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Both non-symbolic and symbolic quantity processing are important for arithmetical computation but not for mathematical reasoning

Yiyun Zhang[a,b,c,d], Chuansheng Chen[e], Hang Liu[c,d], Jiaxin Cui[a,c,d] and Xinlin Zhou[c,a,d]

[a]Advanced Technology Innovation Center for Future Education, Beijing Normal University, Beijing, People's Republic of China; [b]School of Psychology, Liaoning Normal University, Liaoning, People's Republic of China; [c]State Key Laboratory of Cognitive Neuroscience and Learning & IDG/McGovern Institute for Brain Research, Beijing Normal University, Beijing, People's Republic of China; [d]Siegler Center for Innovative Learning, Beijing Normal University, Beijing, People's Republic of China; [e]Department of Psychology and Social Behavior, University of California, Irvine, CA, USA

ABSTRACT
This study investigated whether numerical processing was important for two types of mathematical competence: arithmetical computation and mathematical reasoning. Thousand eight hundred and fifty-seven Chinese primary school children in third through sixth grades took eight computerised tasks: numerical processing (numerosity comparison, digit comparison), arithmetical computation, number series completion, non-verbal matrix reasoning, mental rotation, choice reaction time, and word rhyming. Hierarchical regressions showed that both non-symbolic numerical processing (numerosity comparison) and symbolic numerical processing (digit comparison) were independent predictors of arithmetical computation but neither was a predictor of mathematical reasoning (assessed by number series completion). These findings suggest that the cognitive basis of mathematical performance varies depending on the type of mathematical competence measured.

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Approximate number system; mathematical cognition; arithmetical computation; mathematical reasoning

1. Introduction

Numerical processing is the mental manipulation of quantity information of either symbolic numbers (e.g. Arabic digits) or non-symbolic quantities (e.g. dot arrays, fingers) (Cuneo, 1982; Tudusciuc & Nieder, 2007; Turconi, Jemel, Rossion, & Seron, 2004). The relationship between numerical processing (symbolic or non-symbolic) and mathematical performance has been extensively explored in the field of mathematical cognition, but the results have been somewhat mixed. Many studies have found positive associations between non-symbolic numerical processing and mathematical performance for children (e.g. Bonny & Lourenco, 2013; Butterworth, 2005; Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Landerl, Bevan, & Butterworth, 2004; Landerl & Kölle, 2009; Libertus, Feigenson, & Halberda, 2011, 2013; Mazzocco, Feigenson, & Halberda, 2011; Mundy & Gilmore, 2009; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010). However, other studies did not find significant relationships between non-symbolic numerical processing and mathematical performance (e.g. Fuhs & McNeil, 2013; Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; de Oliveira Ferreira et al., 2012; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Sasanguie, Van den Bussche, & Reynvoet, 2012; Soltész, Szűcs, & Szűcs, 2010; Vanbinst, Ghesquière, & De Smedt, 2012). Similarly, symbolic numerical processing was associated with mathematical performance in many studies when overall reaction time (RT) on the symbolic comparison task was used (e.g. Bugden & Ansari, 2011; Castronovo & Göbel, 2012; De Smedt, Verschaffel, & Ghesquière, 2009; Durand, Hulme, Larkin, & Snowling, 2005; Holloway & Ansari, 2009; Kolkman et al., 2013; Landerl et al., 2004; LeFevre et al., 2010; Mundy & Gilmore, 2009; Rousselle & Noel, 2007; Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013; Sasanguie, Van den Bussche, et al., 2012; Vanbinst

CONTACT Xinlin Zhou zhou_xinlin@bnu.edu.cn

et al., 2012), but not when the distance effects were examined (e.g. Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013).

Because most previous studies did not distinguish among different mathematical domains such as computation, mathematical concepts, and mathematical problem-solving (e.g. Bonny & Lourenco, 2013; Fuhs & McNeil, 2013; Halberda et al., 2008; Libertus et al., 2011, 2013; Mazzocco et al., 2011; Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013; Sasanguie, Van den Bussche, et al., 2012; Soltész et al., 2010), it is not clear whether the mixed results were due to domain differences. The question remains as to whether numerical processing plays an important role in all mathematical domains or only in certain types of mathematical performance. This question is important for both theoretical and practical reasons. Theoretically, this question deals with the cognitive underpinnings of mathematical performance. Results of previous studies have been inconsistent in terms of the role of numerical processing in mathematical performance. To clarify that literature, we believe that it is important to distinguish between computation and reasoning, which are two major components of mathematical abilities. Previous research indeed has shown that they are distinct from each other (e.g. Liang et al., 2007; Mayer, Tajika, & Stanley, 1991; Wei, Yuan, Chen, & Zhou, 2012). Computation emphasises a given set of arithmetic operations on numerals, but mathematical reasoning focuses on searching for higher-order relations among numbers. Numerical processing may play differential roles in arithmetical computation and mathematical reasoning. In terms of practical significance, mathematics education and intervention should consider different cognitive strategies to promote the development of different areas of mathematics (Park & Brannon, 2013).

1.1. Non-symbolic quantity processing

Evidence for the important role of non-symbolic quantity processing in mathematical performance first came from studies on children who suffered from dyscalculia (e.g. Butterworth, 2005; Iuculano, Tang, Hall, & Butterworth, 2008; Landerl et al., 2004). Such children have deficits in both exact numerical representations (e.g. Butterworth, 2005; Landerl et al., 2004) and the approximate number system or ANS (e.g. Feigenson, Dehaene, & Spelke, 2004; Halberda et al., 2012; Halberda et al., 2008; Mussolin, De Volder, et al., 2010; Piazza et al., 2010). For example, Piazza et al. (2010) observed that the ability for numerosity comparison (a common measure of ANS) was severely impaired in 10-year-old children with dyscalculia, and their scores (i.e. Weber fraction) on the non-symbolic quantity processing task were equal to those of 5-year-old typically developing children.

ANS has also been linked to individual differences in mathematical performance in typically developing children (e.g. Bonny & Lourenco, 2013; Halberda et al., 2012; Halberda et al., 2008; Inglis et al., 2011; Libertus et al., 2011, 2013; Mazzocco et al., 2011; Mundy & Gilmore, 2009). For example, Halberda et al. (2008) found that 14-year-old children's performance on a dot comparison task (indexed by Weber fraction) was correlated with scores on standardised mathematics achievement tests (Woodcock–Johnson calculation subtest and Test of Early Mathematics Ability). Inglis et al. (2011) also found a significant correlation between the Weber fraction of numerosity comparison and scores on the calculation subtest of the Woodcock Johnson III Tests of Achievement for children aged 7.6–9.4 years, although the correlation was not significant for adults aged 18–48 years. The role of numerosity comparison in mathematical performance has even been observed in preschoolers. Specifically, Libertus et al. (2011) and Mazzocco et al. (2011) found that preschoolers' ANS performance was significantly correlated with their mathematical ability (measured by the Test of Early Mathematics Ability – Third Edition or TEMA-3), even after controlling for age and verbal skills.

Researchers have not agreed on the underlying mechanism for the close relation between ANS and mathematical performance. There are domain-specific and domain-general explanations. Domain-specific explanations emphasise the common quantity processing of the ANS system and symbolic numerical processing (e.g. Dehaene, Dehaene-Lambertz, & Cohen, 1998; Gallistel & Gelman, 2000; Gilmore, McCarthy, & Spelke, 2007; Libertus, Odic, & Halberda, 2012; Lyons & Beilock, 2011). For example, because of the shared quantity processing, ANS is important for the acquisition of symbolic numerical skills such as counting and arithmetic (e.g. Dehaene et al., 1998; Gallistel & Gelman, 2000; Gilmore et al., 2007). Similarly, ordinality is central to both ANS (e.g. Libertus et al., 2012) and the processing of relations among Arabic numerals (e.g. Libertus et al., 2012; Lyons & Beilock, 2011).

Domain-general explanations have emphasised either the role of inhibitory control in both ANS and mathematical performance (Fuhs & McNeil, 2013; Gilmore et al., 2013) or the role of visual processing in both (Zhou & Cheng, 2015; Zhou, Wei, Zhang, Cui, & Chen, 2015). For example, Zhou et al. (2015) found that the correlation between ANS precision and computation fluency disappeared after controlling for the scores on a geometric figure matching task measuring visual processing.

Not all studies, however, have found significant correlations between ANS precision and children's mathematical performance (e.g. de Oliveira Ferreira et al., 2012; Fuhs & McNeil, 2013; Holloway & Ansari, 2009; Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013; Sasanguie, Van den Bussche, et al., 2012; Soltész et al., 2010; Vanbinst et al., 2012). For example, in a study of kindergartners and first, second, and sixth graders, Sasanguie, De Smedt, et al. (2012) did not find significant correlations between numerosity processing and a curriculum-based standardised achievement test that included 60 items covering number knowledge, understanding of operations, arithmetic, word problem-solving, measurement, and geometry. Vanbinst et al. (2012) also found no relation between non-symbolic numerical processing and general mathematics achievement (multi-digit calculation, word problem-solving, and geometry).

Although the results as reviewed above were mixed, a closer examination of the literature appeared to show a pattern. That is, the studies that showed non-significant results between ANS precision and mathematical performance (e.g. Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013; Vanbinst et al., 2012; Wei, Yuan, et al., 2012) typically used measures of mathematical performance that were beyond computation.

1.2. Symbolic numerical quantity processing

Because mathematics is built on symbolic systems, the connection between symbolic numerical processing and mathematics performance seems self-evident. Evidence for the important role of symbolic numerical quantity processing in mathematics also first came from studies on children who suffered from dyscalculia (e.g. De Smedt, Reynvoet, et al., 2009; Landerl et al., 2004; Landerl & Kölle, 2009; Rousselle & Noel, 2007). Landerl et al. (2004) found that children with dyscalculia had a deficit in their processing of symbolic numbers as well as numerosity (counting), even though these children had higher-than-average IQ, vocabulary, and working memory. Rousselle and Noel (2007) also found that children with mathematical disabilities differed from typically developing children in their symbolic numerical processing. They emphasised the importance of accessing number meaning in the development of mathematical ability. De Smedt, Reynvoet, et al. (2009) explored the correlations between basic number skills (number comparison and number reading) and single-digit arithmetic performance in children with Velo-Cardio-Facial Syndrome. Their results showed that these children's impairment in number comparison was correlated with their poor performance in single-digit computation.

Similar results have been obtained from typically developing children in both cross-sectional studies (e.g. Bugden & Ansari, 2011; Castronovo & Göbel, 2012; Durand et al., 2005; Holloway & Ansari, 2009; Kolkman et al., 2013; Mundy & Gilmore, 2009; Sasanguie, Van den Bussche, et al., 2012; Vanbinst et al., 2012) and longitudinal studies (e.g. De Smedt, Verschaffel, et al., 2009; LeFevre et al., 2010; Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013).

Nevertheless, the above studies typically focused on arithmetical computation (De Smedt, Reynvoet, et al., 2009; Rousselle & Noel, 2007; Zhou et al., 2015). It is thus unclear whether symbolic numerical quantity processing is related to mathematical processing beyond arithmetical computation.

1.3. The current investigation

Based on the above review of the literature, both symbolic and non-symbolic numerical quantity processing are important for arithmetical computation, but their role in mathematics beyond arithmetical computation (e.g. mathematical reasoning and concepts), if any, is not clear. Like arithmetical computation, mathematical reasoning is an important aspect of children's mathematical competence. Mathematical reasoning, also referred to as mathematical problem-solving (Wechsler, 2001), typically involves the following steps to reach resolutions to mathematical problems: understanding the problem, devising a plan, carrying out the plan, and reviewing (Polya, 1957). Unlike arithmetic computation that relies on the retrieval of arithmetic facts or the application of routine procedures, mathematical reasoning often relies on trial and error in the search for answers.

Previous studies have shown a dissociation between arithmetical computation and mathematical reasoning (e.g. Mayer et al., 1991; Nunes, Bryant, Barros, & Sylva, 2012; Wei, Yuan, et al., 2012). For example, Nunes et al. (2012) found that mathematical reasoning and computation made independent contributions to overall mathematical achievement. In their study, mathematical achievement was assessed by two standardised tests, referred to as Key Stage Assessments, which were designed by the UK government. The first assessment was Key Stage 2 (KS2) for sixth graders and the second assessment was Key Stage 3 (KS3) for ninth graders. Both tests cover a variety of aspects of mathematics and are considered as ecologically valid measures of mathematical achievement (Nunes et al., 2012). As for other tests, Nunes et al. used WISC's Arithmetic subtest (WISC-III, Wechsler, 1992) to assess the computational ability, and the Test of Mathematical Reasoning (Nunes, Campos, Magina, & Bryant, 2001) to assess mathematical reasoning. The latter test requires very simple arithmetical computations but makes clear demands on relational reasoning (e.g. "To make 4 good pancakes, you need 4 spoons of flour and 6 of milk. If you want to make 10 good pancakes, how much flour do you need?"). A recent study of mathematical performance and its cognitive correlates (Wei, Yuan, et al., 2012) also found that arithmetical computation and mathematical reasoning (as measured by number series completion) were not significantly correlated. There is also indirect evidence from a cross-cultural study supporting the distinction between computation and mathematical problem-solving in children. Mayer et al. (1991) found that American fifth-grade students had an advantage in arithmetic word problem reasoning, whereas their Japanese counterparts had an advantage in computation.

The current study examined directly the roles of symbolic and non-symbolic numerical quantity processing in arithmetical computation and mathematical reasoning in a large sample of Chinese children. It was hypothesised that both non-symbolic and symbolic numerical quantity processing would be important for arithmetic computation, but not for mathematical reasoning. The rationales for the hypotheses are as follows. First, non-symbolic numerical quantity processing includes both domain-specific and -general processes (e.g. counting, ordinality, form perception, inhibition) that are shared with arithmetical computation. These processes are likely to play only a very small role in mathematical reasoning, for which spatial processing, working memory, and reading are more important (Salthouse & Mitchell, 1990; Wei, Yuan, et al., 2012). Second, as mentioned earlier, symbolic numerical quantity processing is the basis of arithmetical computation, but it plays only a small part in mathematical reasoning.

In the current study, we used the number series completion task to assess mathematical reasoning. Number series completion is one specific aspect of mathematical reasoning that focuses on a search (typically via inductive reasoning) for rules hidden in number series. In several previous studies, the number series completion task was included as an important part of measures of mathematical abilities (e.g. Inglis et al., 2011; Wei, Yuan, et al., 2012; Woodcock, McGrew, & Mather, 2001). For example, it is a subset of Woodcock–Johnson III math achievement test used to measure mathematical reasoning. This task has also been used as a part of some intelligence batteries (e.g. Hayslip, Maloy, & Kohl, 1995; Redick, Unsworth, Kelly, & Engle, 2012) or inductive reasoning (e.g. Holzman, Pellegrino, & Glaser, 1983; Holzman, Pelligrino, & Glaser, 1982; Jia et al., 2011; Liang et al., 2007), both of which are highly correlated with mathematical performance (Haverty, Koedinger, Klahr, & Alibali, 2000). Another reason to use the number series completion as a measure of mathematical reasoning is that both this test and that for arithmetic computation use only Arabic digits to avoid potential confounds such as verbal materials.

2. Method

2.1. Participants

A total of 1857 children (aged from 8.7 to 11.6 years) from third to sixth grade from 14 primary schools in the greater Beijing area took computerised tests of cognitive abilities. One class was randomly selected per grade per school. Table 1 shows participants' information. This study was approved by the Institutional Review Board of Beijing Normal University, the administrators of the departments of education of the relevant counties, and the principals of the schools.

2.2. Tasks

All the tasks were programmed using Web-based applications and are available at www.dweipsy.

Table 1. Participants' information.

		Boys			Girls		
Grade	*N*	*N*	Age (month)	SD	*N*	Age (month)	SD
3	423	235	105.2	4.5	188	104.3	4.9
4	563	306	116.3	4.3	257	116.4	5.1
5	404	211	127.8	5.1	193	127.5	4.8
6	467	244	139.2	19.8	223	137.5	9.6

com/lattice (Wei, Lu, et al., 2012; Wei, Yuan, et al., 2012). Their reliability ranged from .71 to .96 (Wei, Lu, et al., 2012; Wei, Yuan, et al., 2012). Adjusted scores (total correct responses minus total incorrect responses) were used to reduce the effect of guessing in time-limited tasks (e.g. Cirino, 2011; Hedden & Yoon, 2006; Massa & Mayer, 2006; Mayer & Massa, 2003; Wei, Lu, et al., 2012; Wei, Yuan, et al., 2012).

2.2.1. Numerosity comparison

This task, adapted from the Test of Early Mathematics Ability-2 (Ginsburg & Baroody, 1990), was used to assess the ability of processing non-symbolic quantity. In this task, the dots in dot arrays were black and presented within a grey circle with a black background. Participants were asked to judge which dot array contained more dots. They responded by pressing "Q" with their left forefinger if the left array contained more dots, or pressing "P" with their right forefinger if the right side had more dots. The number of dots in each dot array varied from 5 to 12. The ratios of dot arrays (the number of dots in the larger arrays over the number of dots in the smaller arrays) ranged from 1.3 to 1.5. Each pair of dot arrays was presented on the screen until participants responded by pressing a key or until 5000 ms lapsed. After each response, there was a blank of 1000 ms. This test included 36 trials and was allotted 3 min.

We constructed the non-symbolic stimuli for the numerosity comparison task with three rules. First, the total surface area of all dots in a dot array systematically varied: That is, the ratios of total surface area of all dots between the dot arrays with smaller number of dots and those with larger number of dots were 2:1 (incongruent trials), 1:1 (neutral trials), and 1:2 (congruent trials) (see Figure 1). These ratios were the same as those used by Girelli, Lucangeli, and Butterworth (2000). Second, the size of dots (i.e. the diameter of each dot) in a dot array varied pseudo-randomly. Third, the dots in a dot array were pseudo-randomly distributed within a circle.

In the design of this task, five visual properties needed to be taken into account: total surface area, envelope area or convex hull, item size, density (envelope area divided by total surface), and circumference (Gebuis & Reynvoet, 2011). We controlled systematically the total surface area. Since the dots in a dot array were pseudo-randomly distributed within a circle, the envelope area or convex hull was controlled. That is, the difference in envelope areas of two dot arrays was not correlated with their numerical distance ($r = .19$, $p = .27$) or their ratio ($r = .22$, $p = .19$). The item size, density, and circumference were strongly affected by the total surface area, so they had very high correlations with the latter, .97, −.91, and .97, respectively. These coefficients were calculated based on differences in these visual properties between pairs of dot arrays. Therefore, these properties were controlled because the total surface area was controlled.

2.2.2. Digit comparison

This task was adapted from Butterworth's Dyscalculia Screener (Butterworth, 2003). It was used to assess the ability of processing symbolic numerical quantity. In this task, a series of 28 pairs of Arabic numbers (ranging from 2 to 9) in white colour were presented on a black screen in random order. The numerical distance of all digit pairs ranged from 1 to 7 (e.g. 5 vs. 6, 2 vs. 9). The larger number of a pair of numbers was randomly assigned to the

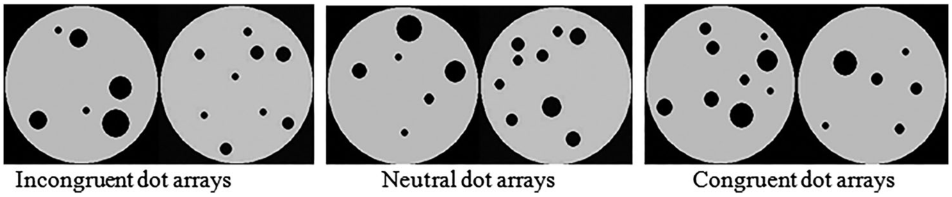

Figure 1. Examples of stimuli for numerosity comparison. The ratios of total surface area of all dots, between the dot arrays with smaller number of dots and those with larger number of dots, are 2:1 (incongruent dot arrays), 1:1 (neutral dot arrays), and 1:2 (congruent dot arrays).

left or right side. The 28 digit pairs were repeated across three conditions, yielding a total of 84 trials. For the congruent condition, one digit was bigger than the other in both numerical quantity and physical size (e.g. 5 vs. 6). For the incongruent condition, one digit was bigger than the other in numerical magnitude but smaller in physical size (e.g. 5 vs. 6). For the neutral condition, digit pairs had the same physical size (e.g. 5 vs. 6). Participants were asked to decide which of the two single-digit numbers was larger in numerical quantity, while ignoring their differences in physical size. They pressed "Q" if the number on the left side was larger or "P" if the number on the right was larger. Due to the large number of trials, this test included three sessions with 28 trials for each session. The three sessions were administered consecutively. The formal test was limited to 3 min.

2.2.3. Simple subtraction

Because simple addition would have been too easy for Chinese middle- to upper-level elementary school students, we used a subtraction task to assess arithmetical computation ability. Participants were given 92 subtraction equations and two alternative answers. The minuends for all problems were smaller than or equal to 18 and the subtrahend was smaller than 10 (e.g. 7–2, 15–8). The incorrect candidate answers were the correct answer plus or minus 1, 2, or 3. Participants were asked to press "Q" with their left forefinger if the answer on the left was correct or "P" with their right forefinger if the answer on the right was correct. The time limit for the formal test was 2 min.

2.2.4. Number series completion

This task was adapted from the Cognitive Abilities Test 3 (Smith, Fernandes, & Strand, 2001). It was used to assess mathematical reasoning. A series of numbers was presented in the middle of the screen, and the participants were asked to judge what the next number would be on the basis of a rule underlying the series of numbers. For example, the series of numbers "1 3 5 7" would have 9 as the next number. Two alternative answers are presented below the given series. Twelve types of rules were involved in the trials (see the Appendix). The occurrence of the rules was random, which should increase the difficulty to find the underlying rule. The participants were asked to press "Q" with the left forefinger if the correct answer was on the left or "P" with the right forefinger if otherwise. The number series and alternative answers remained on the screen until the participants responded. The formal test was limited to 4 min.

2.2.5. Non-verbal matrix reasoning

The task was adapted from Raven's Progressive Matrices test (Raven, 1998). This task was used to control for the influence of general intelligence. There were two candidate answers rather than the original 4–6 candidate answers, because some younger children had difficulty using the mouse or choosing among 4 or 6 keys. Participants were asked to identify the missing segment of a figure according to the figure's inherent regularity. The participants were instructed to press "Q" with their left forefinger if the missing segment was on the left or "P" with their right forefinger if it was on the right. Due to the limited time allotted for this study, we had to shorten the task. The 80 items we used included 44 items from Standard Progressive Matrices (12 items from the first set and 8 items from each one of other four sets) and 36 items from Advanced Progressive Matrices. The formal test was limited to 4 minutes. Shortened forms of this test have been used in previous studies (Bouma, Mulder, & Lindeboom, 1996; Vigneau & Bors, 2001; Vigneau, Caissie, & Bors, 2006). Some studies used the short-form version of the original Raven's Advanced Progressive Matrices test which comprised 14 items from the original 36-item Set II of the APM (Vigneau & Bors, 2001; Vigneau et al., 2006) or the shortened Raven's Standard Progressive Matrices (Bouma et al., 1996) with 36 items rather than 60 items. The split-half reliability of the simplified Raven Progressive Matrices used in the current study was .83 according to our previous study (Wei, Yuan, et al., 2012).

2.2.6. Three-dimensional mental rotation

The three-dimensional mental rotation task was based on Shepard's mental rotation task (Shepard & Metzler, 1971). The task was used to assess spatial processing ability. Studies have showed a close relation between spatial processing and mathematical performance (Berg, 2008; Krajewski & Schneider, 2009; Rohde & Thompson, 2007). The task was used to control for the effect of spatial processing on the relations among quantity processing, arithmetical computation, and mathematical reasoning. In this task, one three-dimensional image was presented on the upper part of the screen and two others on the lower part. Participants were asked to rotate mentally the upper image, and then to choose one of the bottom figures to match the target image after rotation. The angles for rotating images were 15°, 30°, … 345°, with a step of 15°. This task included 180 trials. The formal test was limited to 3 min. Each trial remained on the screen until the participants responded by pressing either "Q" with their left forefinger or "P" with their right forefinger to indicate their choice.

2.2.7. Choice RT

The basic RT task was used to control for the effect of manual response and general processing speed. We modified the simple RT task from Butterworth's Dyscalculia Screener (2003) to match all other tasks in terms of bimanual responses. In this task, a white dot was presented on a black screen, either on the left or right side of "+". The participants were asked to press "Q" with their left forefinger if the dot appeared on the left side of "+"or "P" with their right forefinger if it appeared on the right side of "+". The position where the stimulus occurred on the screen was within 15° of visual angles. The interstimulus interval was randomly determined between 1500 and 3000 ms. This test had 30 trials (15 trials for each side of "+").

2.2.8. Word rhyming

Previous studies have found that language processing is involved in arithmetical computation (e.g. Hecht, Torgesen, Wagner, & Rashotte, 2001; Koponen, Aunola, Ahonen, & Nurmi, 2007; Wei, Yuan, et al., 2012). The word-rhyming task was used to control for the influence of language processing when we examined the relations between quantity processing (especially symbolic numerical processing) and mathematical performance. This task was similar to that used by Tan et al. (2001, 2003) and has been used to assess the ability of phonological processing. Two Chinese characters were presented simultaneously on the screen. The participants had to judge if the two characters rhyme, and to press "Q" with the left forefinger for rhyming pairs (e.g. "门", "人") or "P" with the right forefinger for non-rhyming pairs (e.g. "不", "各"). The stimuli remained on the screen until the participants responded or after a lapse of 4 s. This test had 40 trials. Participants were asked to complete all trials.

2.3. Procedure

Students in the same class took the eight computerised tests together in a computer classroom monitored by six to seven experimenters, with each experimenter monitoring four to six children. The participants were given instructions and practice trials before each task. All children took the eight tasks in a fixed order (choice RT, numerosity comparison, arithmetical computation, digit comparison, mental rotation, word rhyming, number series completion, and non-verbal matrix reasoning). We used a fixed order to allow for group administration of the tests and an examination of individual differences without the confound of the order effect. In addition, we arranged the tasks from easy to difficult to help children to adapt to the tasks. To minimise the fatigue effect, the formal testing sessions were limited to about 30 min and separated by 10-minute resting periods. Participants' responses were automatically recorded in the computer and sent over the Internet to a central server in the laboratory. All data were collected from 12 November to 24 December 2009.

For each task, the participants as a group were first given instruction and practice trials before the formal testing. In the practice session, if the participant made the right choice, the message "Correct! Can you go faster?" would appear in the middle of the screen. If the participant made a mistake, the message "Wrong! Please try again". would flash. There were four or six trials in the practice session, which had the same format as those used in the formal testing. The children were told that they could ask experimenters any questions about the test during the practice session and were forbidden to ask any question during the formal testing. After all the participants completed the practice session for a given task, the formal testing began. When the principal experimenter said "Start", all participants pressed any key to begin the particular formal test. Participants were asked to respond as quickly and accurately as possible, but they were not told the specific amount of time allotted to each task. After all children in a classroom completed a task, they then went on to the next task.

2.4. Data analysis

Hierarchical regression was used to investigate the independent contribution of numerical processing to arithmetical computation and mathematical reasoning. For each dependent variable, two models were run. In the first model, basic cognitive processing (non-verbal matrix reasoning, mental rotation, choice reaction, and word rhyming), gender, and age were entered in the first step. Numerosity comparison was then entered in the second step, and finally digit comparison was entered in the third step. The second model reversed the second and third steps of the first model. We then directly compared their explained variance across different grade levels in order to examine the grade-related developmental differences in the role of the two types of numerical processing in computation.

We also conducted mediation analyses with the bootstrapping method (Preacher & Hayes, 2008) to quantify the differential contributions of symbolic and non-symbolic processing to mathematical performance after controlling for general cognitive processing.

3. Results

The mean scores and standard deviations for all eight tasks are displayed by grade level in Table 2. The intercorrelation coefficients of all measures for the total sample are shown in Table 3. Twenty-four hierarchical regression analyses (2 dependent measures × 4 grade levels × 3 types of step combinations) were conducted to investigate the independent contribution of numerical processing to arithmetical computation and mathematical reasoning. Results are shown in Tables 4–6. Another set of 24 hierarchical regression analyses was conducted to examine whether numerosity comparison in the 3 conditions (congruent, neutral, and incongruent) predicted arithmetical computation (see Table 7). With Bonforroni correction, the adjusted alpha of .05 corresponded to .001 before correction.

Results showed that basic cognitive processing (i.e. non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming) accounted for between 18% and 34% of the variance of mathematical performance (see Table 4). As shown in Tables 5 and 6, after controlling for age, gender, and cognitive processing, numerosity comparison accounted for a significant portion of the variance of arithmetical computation (7.5% for third grade; 5.7% for fourth grade; 4.8% for fifth grade; and 5.1% for sixth grade) (see Table 5). After the addition of digit comparison as a control variable, numerosity comparison still accounted for a significant portion of the variance of arithmetical computation, 5.4% for third grade, 4.4% for fourth grade, 3.1% for fifth grade, and 2.6% for sixth grade (see Table 6). Numerosity comparison was not a predictor of mathematical reasoning for any of the grade levels.

Digit comparison was a consistent predictor of arithmetical computation for all grades after controlling for age, gender, and cognitive processing, 4.7% for third grade, 5.2% for fourth grade, 5.2% for fifth grade, and 7.3% for sixth grade (see Table 6). After the addition of numerosity comparison as a control variable, digit comparison still accounted for a significant portion of the variance of arithmetical computation 2.6% for third grade, 3.9% for fourth grade, 3.5% for fifth grade, and 4.8% for sixth grade (see Table 5). Digit comparison was not a predictor of mathematical reasoning. For third grade, however, digit comparison RT significantly predicted mathematical reasoning after controlling for gender, age and basic cognitive processing (3.4%). Surprisingly, the relation was positive rather than negative.

Table 2. Means and standard deviations of all the measures for the eight tasks by grade level.

	Grade 3		Grade 4		Grade 5		Grade 6	
Tasks	*M*	SD	*M*	SD	*M*	SD	*M*	SD
Numerosity comparison (accuracy rate)	.84	.10	.86	.09	.88	.08	.88	.07
Numerosity comparison (reaction time)	1027	249	926	206	897	187	822	178
Digit comparison (accuracy rate)	.90	.06	.90	.06	.92	.05	.93	.05
Digit comparison (reaction time)	848	163	766	149	734	127	672	109
Arithmetical computation	35.0	9.8	37.6	10.2	40.7	8.0	42.8	8.3
Number series completion	7.2	6.80	8.3	6.4	10.2	6.3	11.2	6.4
Non-verbal matrix reasoning	12.6	9.5	14.9	8.8	15.7	8.6	17.1	8.8
Mental rotation	11.9	9.8	14.8	10.5	17.0	10.7	18.4	10.4
Choice reaction time (accuracy rate)	.95	.08	.95	.10	.96	.08	.97	.07
Choice reaction time (reaction time)	543	148	473	154	467	186	417	134
Word rhyming (accuracy rate)	.68	.17	.74	.17	.81	.14	.82	.15
Word rhyming (reaction time)	2620	730	2455	733	2494	581	2370	641

Note: The unit for RT is millisecond. For all the time-limited tasks including arithmetical computation, number series completion, non-verbal matrix reasoning and mental rotation, the measure is the adjusted number of correct trials.

Table 3. Intercorrelations (Spearman) of all measures.

Task	1	2	3	4	5	6	7	8	9	10	11
1.Numerosity comparison (accuracy rate)	–	–	–	–	–	–	–	–	–	–	–
2.Numerosity comparison (reaction time)	.23*	–	–	–	–	–	–	–	–	–	–
3.Digit comparison (accuracy rate)	.38*	.19*	–	–	–	–	–	–	–	–	–
4.Digit comparison (reaction time)	.05	.56*	.14*	–	–	–	–	–	–	–	–
5.Arithmetical computation	.30*	−.12*	.31*	−.31*	–	–	–	–	–	–	–
6.Number series completion	.23*	−.03	.27*	−.06	.46*	–	–	–	–	–	–
7.Non-verbal matrix reasoning	.23*	−.05	.21*	−.08*	.33*	.40*	–	–	–	–	–
8.Mental rotation	.24*	−.05	.24*	−.09*	.30*	.32*	.37*	–	–	–	–
9.Choice reaction time (accuracy rate)	.23*	.12	.26*	.04	.16*	.17*	.14*	.11*	–	–	–
10.Choice reaction time (reaction time)	.05	.53*	.07	.54*	−.32*	−.19*	−.18*	−.17*	.07	–	–
11.Word rhyming (accuracy rate)	.24*	−.17*	.31*	−.24*	.53*	.50*	.36*	.30*	.17*	−.34*	–
12.Word rhyming (reaction time)	.11*	.24*	.17*	.30*	−.10*	.03	.03	.06	.04	.23*	−12*

Note: *$p < .05$, corrected with Bonferroni correction method among all the correlation.

That is, larger RT was associated with better scores on the mathematical reasoning test.

It should be noted that the variances were not equal for the two mathematical tests (arithmetical computation and mathematical reasoning) used in the current investigation. We have calculated the coefficients of variation (CV) for both arithmetical computation and mathematical reasoning using the equation: CV = standard deviation/mean. The variances were larger for mathematical reasoning than for computation for all four grade levels. These differences, however, could not have

Table 4. Hierarchical regression models predicting arithmetical computation and number series completion from age, gender (Step 1), and general cognitive processing (Step 2).

		Arithmetical computation		Mathematical reasoning	
Grade	Predictors	Step 1 B (SE)	Step 2 B (SE)	Step 1 B (SE)	Step 2 B (SE)
3	Age	−.08 (.10)	−.07 (.09)	.06 (.07)	.07 (.07)
	Gender	.54 (.96)	−.09 (.85)	.97 (.67)	.65 (.62)
	Non-verbal matrix reasoning	–	.16 (.05)	–	.14 (.03)*
	Mental rotation	–	.17 (.05)*	–	.09 (.03)
	Choice reaction (accuracy rate)	–	−1.26 (5.09)	–	.17 (3.75)
	Choice reaction (reaction time)	–	−.01 (.00)	–	.00 (.00)
	Word rhyming (accuracy rate)	–	17.04 (2.70)*	–	8.93 (1.99)*
	Word rhyming (reaction time)	–	.00 (.00)	–	.00 (.00)
		$R^2 = .002$	$\Delta R^2 = .266$*	$R^2 = .006$	$\Delta R^2 = .175$*
4	Age	−.04 (.01)*	−.03 (.01)	.00 (.01)	.01 (.01)
	Gender	.95 (.86)	.15 (.80)	.92 (.54)	.39 (.49)
	Non-verbal matrix reasoning	–	.18 (.05)*	–	.15 (.03)*
	Mental rotation	–	.10 (.04)	–	.10 (.03)*
	Choice reaction (accuracy rate)	–	6.72 (3.72)	–	5.35 (2.28)
	Choice reaction (reaction time)	–	−.00 (.00)	–	.00 (.00)
	Word rhyming (accuracy rate)	–	17.94 (2.56)*	–	9.47 (1.57)*
	Word rhyming (reaction time)	–	.00 (.00)	–	.00 (.00)
		$R^2 = .024$	$\Delta R^2 = .186$*	$R^2 = .005$	$\Delta R^2 = .237$*
5	Age	−.13 (.08)	−.10 (.07)	.01 (.06)	.02 (.06)
	Gender	2.49 (.78)	.81 (.76)	1.50 (.63)	.62 (.60)
	Non-verbal matrix reasoning	–	.14 (.05)	–	.19 (.04)*
	Mental rotation	–	.02 (.04)	–	.06 (.03)
	Choice reaction (accuracy rate)	–	8.59 (6.07)	–	.60 (4.75)
	Choice reaction (reaction time)	–	.00 (.00)	–	.00 (.00)
	Word rhyming (accuracy rate)	–	18.36 (2.77)*	–	13.48 (2.17)*
	Word rhyming (reaction time)	–	−.00 (.00)	–	−.00 (.00)
		$R^2 = .031$	$\Delta R^2 = .183$*	$R^2 = .014$	$\Delta R^2 = .228$*
6	Age	.02 (.02)	−.00 (.02)	.04 (.02)	.02 (.02)
	Gender	1.04 (.77)	.31 (.66)	.84 (.59)	.19 (.50)
	Non-verbal matrix reasoning	–	.07 (.04)	–	.18 (.03)*
	Mental rotation	–	.11 (.04)	–	.07 (.03)
	Choice reaction (accuracy rate)	–	4.04 (4.96)	–	3.78 (3.73)
	Choice reaction (reaction time)	–	−.01 (.00)	–	−.00 (.00)
	Word rhyming (accuracy rate)	–	20.77 (2.51)*	–	14.14 (1.89)*
	Word rhyming (reaction time)	–	.00 (.00)	–	.00 (.00)
		$R^2 = .006$	$\Delta R^2 = .302$*	$R^2 = .015$	$\Delta R^2 = .335$*

Note: *$p < .05$, corrected with Bonferroni correction method among all the regression analyses (in Table 4–7).

Table 5. Hierarchical regression models predicting arithmetical computation and number series completion from number comparison after controlling for numerosity comparison and other factors (including age and gender in Step 1 and general cognitive processing in Step 2, as shown in Table 4).

Grade	Predictors	Arithmetical computation Step 3 B (SE)	Arithmetical computation Step 4 B (SE)	Number series completion Step 3 B (SE)	Number series completion Step 4 B (SE)
3	Age	−0.14 (.09)	−.12 (.08)	.08 (.07)	.08 (.07)
	Gender	0.29 (.81)	.54 (.80)	.74 (.63)	.48 (.62)
	Non-verbal matrix reasoning	0.11 (.05)	.11 (.04)	.14 (.04)*	.14 (.03)*
	Mental rotation	.14 (.04)	.12 (.04)	.09 (.03)	.07 (.03)
	Choice reaction (accuracy rate)	−5.73 (4.90)	−5.82 (4.88)	−.37 (3.80)	−2.89 (3.78)
	Choice reaction (reaction time)	−.01 (.00)*	−0.01 (.00)	−.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	14.64 (2.59)*	13.05 (2.57)*	8.86 (2.01)*	9.34 (1.99)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Numerosity comparison (accuracy rate)	31.44 (4.59)*	26.99 (4.64)*	1.43 (3.56)	3.04 (3.60)
	Numerosity comparison (reaction time)	−.00 (.00)	−.00 (.00)	.00 (.00)	−.00 (.00)
	Digit comparison (accuracy rate)	–	25.43 (7.38)	–	5.23 (5.71)
	Digit comparison (reaction time)	–	−.01 (.00)	–	.01 (.00)*
		$\Delta R^2 = .075$*	$\Delta R^2 = .026$*	$\Delta R^2 = .002$	$\Delta R^2 = .034$*
4	Age	−.03 (.01)	−.03 (.01)	.01 (.01)	.01 (.01)
	Gender	.16 (.77)	.32 (.76)	.47 (.49)	.33 (.49)
	Non-verbal matrix reasoning	.16 (.05)	.16 (.05)	.14 (.03)*	.14 (.03)*
	Mental rotation	.08 (.04)	.06 (.04)	.10 (.03)*	.09 (.03)*
	Choice reaction (accuracy rate)	4.19 (3.63)	4.39 (3.55)	4.68 (2.30)	4.27 (2.29)
	Choice reaction (reaction time)	−.00 (.00)	−.00 (.00)	.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	15.00 (2.53)*	12.58 (2.52)*	9.76 (1.60)*	9.13 (1.63)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Numerosity comparison (accuracy rate)	30.72 (4.81)*	25.94 (4.82)*	−.13 (3.04)	−1.50 (3.12)
	Numerosity comparison (reaction time)	−.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	–	21.26 (7.12)	–	10.20 (4.60)
	Digit comparison (reaction time)	–	−.02 (.00)*	–	.00 (.00)
		$\Delta R^2 = .057$*	$\Delta R^2 = .039$*	$\Delta R^2 = .010$	$\Delta R^2 = .008$
5	Age	−.11 (.07)	−.14 (.07)	.01 (.06)	.01 (.06)
	Gender	.86 (.76)	.85 (.74)	.58 (.61)	.52 (.61)
	Non-verbal matrix reasoning	.14 (.04)	.12 (.04)	.18 (.04)*	.18 (.04)*
	Mental rotation	.00 (.04)	.00 (.03)	.05 (.03)	.05 (.03)
	Choice reaction (accuracy rate)	.16 (6.32)	−3.43 (6.30)	−1.75 (5.07)	−3.70 (5.15)
	Choice reaction (reaction time)	−.00 (.00)	−.01 (.00)	−.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	16.90 (2.73)*	13.93 (2.75)*	12.85 (2.18)*	12.28 (2.25)*
	Word rhyming (reaction time)	−.00 (.00)	−.00 (.00)	.00 (.00)	.00 (.00)
	Numerosity comparison (accuracy rate)	19.28 (4.93)*	15.68 (4.90)	8.01 (3.95)	6.86 (4.00)
	Numerosity comparison (reaction time)	.01 (.00)	.01 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	–	36.99 (8.83)*	–	11.92 (7.22)
	Digit comparison (reaction time)	–	−.01 (.00)	–	.00 (.00)
		$\Delta R^2 = .048$*	$\Delta R^2 = .035$*	$\Delta R^2 = .010$	$\Delta R^2 = .007$
6	Age	−.01 (.02)	−.00 (.02)	.02 (.02)	.02 (.02)
	Gender	.80 (.65)	.78 (.63)	.37 (.50)	.34 (.50)
	Non-verbal matrix reasoning	.04 (.04)	.04 (.04)	.17 (.03)*	.17 (.03)*
	Mental rotation	.08 (.03)	.06 (.03)	.07 (.03)	.06 (.03)
	Choice reaction (accuracy rate)	2.46 (4.83)	4.84 (4.67)	2.53 (3.74)	2.94 (3.76)
	Choice reaction (reaction time)	−.01 (.00)*	−.01 (.00)*	−.01 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	18.10 (2.46)*	15.52 (2.41)*	13.45 (1.91)*	12.99 (1.94)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	−.00 (.00)	−.00 (.00)
	Numerosity comparison (accuracy rate)	26.21 (4.66)*	18.77 (4.66)*	5.79 (3.61)	4.45 (3.75)
	Numerosity comparison (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	–	39.75 (7.52)*	–	7.89 (6.05)
	Digit comparison (reaction time)	–	−.01 (.00)*	–	−.00 (.00)
		$\Delta R^2 = .051$*	$\Delta R^2 = .048$*	$\Delta R^2 = .012$	$\Delta R^2 = .002$

Note: *$p < .05$, corrected with Bonferroni correction method among all the regression analyses (in Table 4–7).

accounted for the main finding that numerical processing was correlated with arithmetical computation (which has less variance), but not with mathematical reasoning (which has more variance).

Hierarchical regression was also used to examine whether numerosity comparison and digit comparison in three conditions (congruent, neutral, and incongruent) predicted arithmetical computation (see Table 7). The first and second steps were the same as those in Table 4, and thus were not displayed. Table 7 only showed the R^2 change (third step). The neutral condition of numerosity comparison and digit comparison generally predicted arithmetical computation for seven of the eight analyses, except for the neutral condition of numerosity comparison in sixth grade. The congruent condition was

Table 6. Hierarchical regression models predicting arithmetical computation and number series completion from numerosity comparison after controlling for number comparison and other factors (including age and gender in Step 1 and general cognitive processing in Step 2, as shown in Table 4).

Grade	Predictors	Arithmetical computation		Number series completion	
		Step 3 B (SE)	Step 4 B (SE)	Step 3 B (SE)	Step 4 B (SE)
3	Age	−.06 (.09)	−.12 (.08)	.09 (.06)	.08 (.07)
	Gender	.20 (.82)	.54 (.80)	.52 (.61)	.48 (.62)
	Non-verbal matrix reasoning	.15 (.05)	.11 (.04)	.15 (.03)*	.14 (.03)*
	Mental rotation	.14 (.04)	.12 (.04)	.08 (.03)	.07 (.03)
	Choice reaction (accuracy rate)	−1.99 (5.03)	−5.82 (4.88)	−2.52 (3.74)	−2.89 (3.78)
	Choice reaction (reaction time)	−.01 (.00)	−.01 (.00)	−.01 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	14.51 (2.66)*	13.05 (2.57)*	9.47 (1.98)*	9.34 (1.99)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	33.66 (7.53)*	25.43 (7.38)	5.85 (5.60)	5.23 (5.71)
	Digit comparison (reaction time)	−.01 (.00)*	−.01 (.00)	.01 (.00)*	.01 (.00)*
	Numerosity comparison (accuracy rate)	–	26.99 (4.64)*	–	3.04 (3.60)
	Numerosity comparison (reaction time)	–	−.00 (.00)	–	−.00 (.00)
		$\Delta R_2^2 = .047$*	$\Delta R_3^2 = .054$*	$\Delta R_2^2 = .034$*	$\Delta R_3^2 = .002$
4	Age	−.03 (.01)	−.03 (.01)	.01 (.01)	.01 (.01)
	Gender	.11 (.78)	.32 (.76)	.25 (.49)	.33 (.49)
	Non-verbal matrix reasoning	.18 (.05)*	.16 (.05)	.14 (.03)*	.14 (.03)*
	Mental rotation	.07 (.04)	.06 (.04)	.09 (.03)*	.09 (.03)*
	Choice reaction (accuracy rate)	6.36 (3.63)	4.39 (3.55)	4.45 (2.28)	4.27 (2.29)
	Choice reaction (reaction time)	.00 (.00)	−.00 (.00)	.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	13.97 (2.57)*	12.58 (2.52)*	8.83 (1.61)*	9.13 (1.63)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	33.10 (7.00)*	21.26 (7.12)	11.13 (4.40)	10.20 (4.60)
	Digit comparison (reaction time)	−.01 (.00)*	−.02 (.00)*	.00 (.00)	.00 (.00)
	Numerosity comparison (accuracy rate)	–	25.94 (4.82)*	–	−1.50 (3.12)
	Numerosity comparison (reaction time)	–	.00 (.00)	–	.00 (.00)
		$\Delta R_2^2 = .052$*	$\Delta R_3^2 = .044$*	$\Delta R_2^2 = .014$	$\Delta R_3^2 = .004$
5	Age	−.13 (.07)	−.14 (.07)	.02 (.06)	.01 (.06)
	Gender	.76 (.74)	.85 (.74)	.61 (.59)	.52 (.61)
	Non-verbal matrix reasoning	.12 (.04)	.12 (.04)	.18 (.04)*	.18 (.04)*
	Mental rotation	.01 (.04)	.00 (.03)	.05 (.03)	.05 (.03)
	Choice reaction (accuracy rate)	1.02 (6.24)	−3.43 (6.30)	−3.38 (5.01)	−3.70 (5.15)
	Choice reaction (reaction time)	−.00 (.00)	−.01 (.00)	−.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	14.75 (2.79)*	13.93 (2.75)*	12.67 (2.24)*	12.28 (2.25)*
	Word rhyming (reaction time)	−.00 (.00)	−.00 (.00)	.00 (.00)	.00 (.00)
	Digit comparison (accuracy rate)	46.02 (8.73)*	36.99 (8.83)*	14.24 (7.01)	11.92 (7.22)
	Digit comparison (reaction time)	−.00 (.00)	−.01 (.00)	.00 (.00)	.00 (.00)
	Numerosity comparison (accuracy rate)	–	15.68 (4.90)	–	6.86 (4.00)
	Numerosity comparison (reaction time)	–	.01 (.00)	–	.00 (.00)
		$\Delta R_2^2 = .052$*	$\Delta R_3^2 = .031$*	$\Delta R_2^2 = .011$	$\Delta R_3^2 = .006$
6	Age	.00 (.02)	−.00 (.02)	.02 (.02)	.02 (.02)
	Gender	.37 (.63)	.78 (.63)	.17 (.50)	.34 (.50)
	Non-verbal matrix reasoning	.07 (.04)	.04 (.04)	.18 (.03)*	.17 (.03)*
	Mental rotation	.06 (.03)	.06 (.03)	.06 (.03)	.06 (.03)
	Choice reaction (accuracy rate)	6.55 (4.72)	4.84 (4.67)	4.03 (3.74)	2.94 (3.76)
	Choice reaction (reaction time)	−.01 (.00)	−.01 (.00)*	−.00 (.00)	−.00 (.00)
	Word rhyming (accuracy rate)	16.75 (2.44)*	15.52 (2.41)*	13.30 (1.93)*	12.99 (1.94)*
	Word rhyming (reaction time)	.00 (.00)	.00 (.00)	.00 (.00)	−.00 (.00)
	Digit comparison (accuracy rate)	50.47 (7.26)*	39.75 (7.52)*	12.04 (5.75)	7.89 (6.05)
	Digit comparison (reaction time)	−.01 (.00)*	−.01 (.00)*	.00 (.00)	−.00 (.00)
	Numerosity comparison (accuracy rate)	–	18.77 (4.66)*	–	4.45 (3.75)
	Numerosity comparison (reaction time)	–	.00 (.00)	–	.00 (.00)
		$\Delta R_2^{22} = .073$*	$\Delta R_3^2 = .026$*	$\Delta R_2^2 = .007$	$\Delta R_3^2 = .008$

Note: *$p < .05$, corrected with Bonferroni correction method among all the regression analyses (in Table 4–7).

Table 7. The R^2 changes in Step 3 in the hierarchical regression models predicting arithmetical computation from three types of numerical comparison (age and gender in Step 1 and general cognitive processing in Step 2, shown in Table 4).

Grade	Numerosity comparison			Number comparison		
	Congruent	Incongruent	Neutral	Congruent	Incongruent	Neutral
3	.068*	.020	.051*	.019	.021	.058*
4	.035*	.017	.061*	.029*	.034*	.051*
5	.042*	.028	.044*	.055*	.022	.045*
6	.021	.045*	.018	.022	.065*	.038*

Note: *$p < .05$, corrected with Bonferroni correction method among all the regression analyses (in Table 4–7).

also a significant predictor for five of the analyses, and the incongruent condition was a significant predictor for three analyses. Paired *t*-test did not show significant differences in the amounts of explained variances between the neutral and incongruent conditions, $t\ (7) = 1.48$, $p = .182$.

To further quantify the differential contributions of symbolic and non-symbolic processing to arithmetical computation (after controlling for gender, age, and general cognitive processing), mediation analyses were conducted (because hierarchical regression analyses showed that symbolic and non-symbolic processing made no significant contributions to mathematical reasoning, no mediation analysis was conducted for mathematical reasoning). For the mediation analyses, the dependent variable was the residuals of arithmetical computation after controlling for four general cognitive processing variables (non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming) as well as age and gender. We first tested whether numerosity comparison mediated the relation between digit comparison and arithmetical computation and found a partial mediation. The mediation effect accounted for about 25.75% of the total effect (see panel A of Figure 2). We then tested whether digit comparison mediated the relation between numerosity comparison and arithmetical computation. The mediating role of digit comparison was also partial, accounting for about 25.67% of the total effect (see panel B of Figure 2).

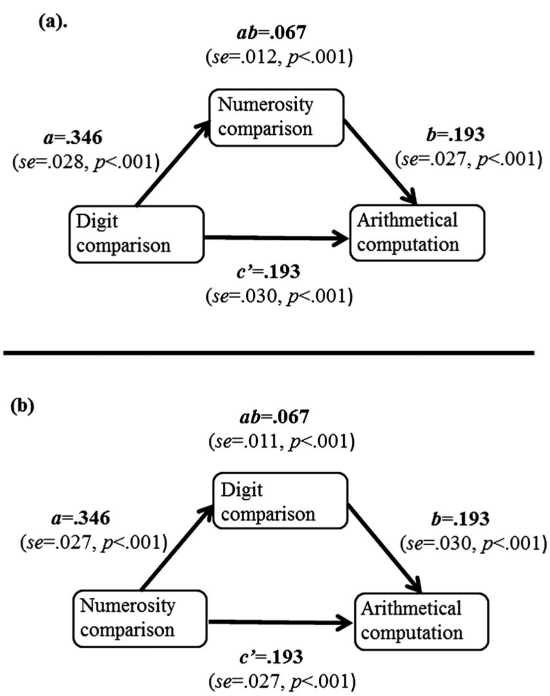

Figure 2. Mediation analyses for the differential contributions of numerosity comparison and digit comparison to arithmetical computation. The top panel (A) is for the mediation effect of numerosity comparison on the relation between digit comparison and arithmetical computation, the bottom panel (B) is for the mediation effect of digit comparison on the relation between numerosity comparison and arithmetical computation. Note: (1). Arithmetical computation refers to the non-standardised residual of arithmetical computation after controlling for general cognitive processing (non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming) as well as age and gender differences. (2). The model is constrained by the assumption of $c = ab + c'$. c: direct effect of the original predictor; ab: indirect effect of the mediator, and c': the remaining (unmediated) direct effect.

4. Discussion

The main goal of the current study was to test whether non-symbolic and symbolic numerical processing similarly contributed to two subtypes of mathematical competence (arithmetical computation and mathematical reasoning) in a large sample of Chinese primary school children. Children from third to sixth grades performed numerosity and digit comparisons, arithmetical computation, number series completion, and other cognitive tasks (i.e. non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming). Hierarchical regression analyses showed that numerosity comparison and digit comparison made independent contributions to arithmetical computation but not to mathematical reasoning, after controlling for gender, age, and scores on non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming.

Cognitive mechanisms involved in mathematical performance have long been a hot topic in the field of mathematical cognition and learning. Cognitive factors have been found to account for a substantial amount of variance in mathematical performance (Praet, Titeca, Ceulemans, & Desoete, 2013; Swanson & Kim, 2007; Zhou et al., 2015). The current study showed that 17–33% variance in arithmetical computation and mathematical reasoning can be accounted for by the four types of general cognitive factors included in the current study, namely, non-verbal matrix reasoning, mental rotation, choice RT, and word rhyming. The explained amount of variance was moderate probably because we did not include many other measures of mathematics-relevant cognitive factors such as working memory and executive function (Fuhs & McNeil, 2013; Gilmore et al., 2013; Swanson & Kim, 2007).

Beyond the basic cognitive factors, the roles of non-symbolic and symbolic quantity processing in mathematical performance have been closely examined (Butterworth, 2005; De Smedt, Janssen, et al., 2009; Halberda et al., 2008; Holloway & Ansari, 2009; Inglis et al., 2011; Landerl et al., 2004; Piazza et al., 2010; Rousselle & Noel, 2007). Previous studies found inconsistent results regarding the relations between non-symbolic numerical quantity processing and mathematical performance, perhaps due to the use of different measures of mathematical performance in different studies. According to the review by De Smedt, Noël, Gilmore, and Ansari (2013), no study has directly tested whether non-symbolic and symbolic numerical processing was important for some or all domains of mathematics for children.

4.1. Relation between numerical quantity processing and arithmetical computation

The ANS is believed to provide a basis for the acquisition of symbolic numerical skills such as counting and arithmetic (Dehaene et al., 1998; Gallistel & Gelman, 2000; Gilmore, McCarthy, & Spelke, 2010; Halberda et al., 2008; Piazza et al., 2010; but see Butterworth, 2010; Noël & Rousselle, 2011). Indeed, some of the previous studies have shown a significant association between ANS precision and arithmetic performance (De Smedt, Reynvoet, et al., 2009; Halberda et al., 2008; Inglis et al., 2011; Mussolin, Mejias, et al., 2010; Piazza et al., 2010; Zhou et al., 2015). This study replicated that finding with a large sample of Chinese primary school children.

We should note that we used the dot arrays with 5–12 dots, which were within the range of Halberda et al.'s (2008) 5–16 dots. The numerosity processing in the current investigation is probably still closer to the approximate representational system than the exact representational system. Nevertheless, the relation found in this study between numerosity comparison and mathematical performance might also involve the exact representational system for the trials containing small dot arrays, such as five dots. Future studies should separate the two types of numerosity processing.

It is worth noting that numerosity comparison and digit comparison made independent contributions to arithmetical computation, suggesting that they play differential roles in computation. For example, counting could be more likely to be involved in dot comparison, whereas the mental number line could be more likely to be involved in in digit comparison and arithmetical computation (Nuerk, Weger, & Willmes, 2001; Yu et al., 2015; Zhou, Zhao, Chen, & Zhou, 2012). Importantly, digit comparison has been considered to be a component of arithmetic (Butterworth, Zorzi, Girelli, & Jonckheere, 2001).

4.2. Relation between numerical quantity processing and mathematical reasoning

We found that non-symbolic and symbolic numerical quantity processing were not significant predictors of children's performance in mathematical reasoning (or number series completion) after controlling for basic cognitive processes and gender. This finding extended the results of previous studies with adults that measured mathematical performance in terms of problem-solving, concepts, geometry, etc. (Sasanguie, De Smedt, et al., 2012; Sasanguie et al., 2013; Vanbinst et al., 2012; Wei, Yuan, et al., 2012). In other words, our result constrained the role of numerical processing to mathematical computation, which involves the retrieval of arithmetic facts and application of routine procedures in the manipulation of numerical quantity. In contrast, mathematical reasoning focuses on the underlying relations among particular sets of numbers. As Geary (1994) described, mathematical reasoning is a domain of mathematics that encompasses mathematical problem-solving skills, which is more than simply applying routine procedures and accessing numerical quantity. Problem-solving is typically based on the four steps: understanding the problem, designing a plan, executing the plan, and reviewing. These steps can be applied to all domains of problem-solving with differential emphases on different steps. In the case of number series completion, the most important step is to search for hidden rules of relations among numbers within the series with inductive reasoning. Numerical quantity processing is typically easy. Thus, we observed that the non-verbal matrix reasoning, rather than numerical quantity processing, was a consistent predictor of mathematical reasoning. Our result suggests that basic cognitive processing is important for mathematical reasoning.

Although the current investigation showed that numerical quantity was not associated with mathematical reasoning as measured with number series completion, we need to note that number series completion is only one aspect of mathematical reasoning (inductive reasoning). Broader mathematical reasoning involves solving real-world, practical problems covering a wide range of subjects, such as time, money, and measurement. Future study is needed to directly test the role of basic numerical quantity processing in real-world mathematical problem-solving.

4.3. Practical implications

The current study found independent contributions of two types of numerical quantity processing (ANS and basic symbolic numerical processing) to arithmetical computation. Previous studies have shown that the approximate and exact number systems can be as targets for effective interventions to promote mathematical learning (Butterworth & Laurillard, 2010; Obersteiner, Reiss, & Ufer, 2013; Park & Brannon, 2013; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). For example, Park and Brannon (2013) trained adults on non-symbolic approximate arithmetic (2 dots arrays with 9–36 dots separately) over the course of 10 training sessions. Results showed improved performance in symbolic mathematics including addition and subtraction. As another example, Butterworth and Laurillard (2010) used Dots2Track program and Dots2Digits program to train children with low numeracy and dyscalculia. The Dots2Track helps the learner discern the relationship between the numerosity in a dot pattern and its representation as a digit on a number line. Dots2Digits helps children pair up a dot pattern with its appropriate digit, and vice versa. They found that training had a positive effect on children's learning and that teachers evaluated the training program positively. One previous study directly compared the training effects of the approximate and exact number systems (Obersteiner et al., 2013) and found that both types of training resulted in improved performance but there was no crossover effect. That is, the approximate instructional approach only improved the task relying on the approximate number representation, and the exact instructional approach only improved the task relying on the exact number representation. As the authors of that study argued, the approximate and exact number systems may rely on distinct cognitive processing, and both instructional approaches are needed.

In sum, the differential roles of numerical processing in different types of mathematics should have important implications for mathematics education and intervention. Previous studies have emphasised non-symbolic and symbolic quantity processing as a target for effective interventions for mathematical learning difficulty (e.g. Butterworth & Laurillard, 2010; Wilson et al., 2006). Our result suggests that, because numerical processing is not important for mathematical reasoning, different strategies are needed to improve mathematical abilities in different domains.

4.4. Conclusion

The main finding of the current investigation was that numerical quantity processing in the approximate and exact number systems was correlated with arithmetical computation but not with mathematical reasoning. This finding is important because it suggests that previous studies might have overemphasised the role of numerical quantity processing in the approximate and exact number systems in the development of mathematical skills. Given the evidence for the limited role of numerical quantity processing in mathematics other than computation, future neuroimaging research should also go beyond the emphasis on numerical quantity processing. Instead, neural bases for basic cognitive processes such as general reasoning and visuospatial processing should be considered when studying mathematical (dis)abilities.

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References

Berg, D. H. (2008). Working memory and arithmetic calculation in children: The contributory roles of processing speed, short-term memory, and Reading. *Journal of Experimental Child Psychology*, *99*(4), 288–308.

Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology*, *114*(3), 375–388.

Bouma, J. M., Mulder, J. L., & Lindeboom, J. (1996). *Neuropsychologische Diagnostiek: Handboek*. Lisse: Swets & Zeitlinger.

Bugden, S., & Ansari, D. (2011). Individual differences in children's mathematical competence are related to the intentional but not automatic processing of arabic numerals. *Cognition*, *118*(1), 32–44.

Butterworth, B. (2003). *Dyscalculia screener*. London: nferNelson.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry*, *46* (1), 3–18.

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences*, *14*(12), 534–541.

Butterworth, B., & Laurillard, D. (2010). Low numeracy and dyscalculia: Identification and intervention. *ZDM*, *42*(6), 527–539.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. (2001). Storage and retrieval of addition facts: The role of number comparison. *The Quarterly Journal of Experimental Psychology: Section A*, *54*(4), 1005–1029.

Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PLoS One*, *7*(4), e33832.

Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, *148*, 163–172.

Cirino, P. T. (2011). The interrelationships of mathematical precursors in kindergarten. *Journal of Experimental Child Psychology*, *108*(4), 713–733.

Cuneo, D. O. (1982). Children's judgments of numerical quantity: A new view of early quantification. *Cognitive Psychology*, *14*(1), 13–44.

De Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first grade to second grade. *Journal of Experimental Child Psychology*, *103*(2), 186–201.

De Smedt, B., Noël, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*(2), 48–55.

De Smedt, B., Reynvoet, B., Swillen, A., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Basic number processing and difficulties in single-digit arithmetic: Evidence from velo-cardio-facial syndrome. *Cortex*, *45*(2), 177–188.

De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, *103*(4), 469–479.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences*, *21*(8), 355–361.

Durand, M., Hulme, C., Larkin, R., & Snowling, M. (2005). The cognitive foundations of Reading and arithmetic skills in 7-to 10-year-olds. *Journal of Experimental Child Psychology*, *91*(2), 113–136.

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, *123*, 53–72.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314.

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: Contributions of inhibitory control. *Developmental Science*, *16*(1), 136–148.

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, *4*(2), 59–65.

Geary, D. C. (1994). Children's mathematical development: Research and practical applications. *Washington, LAnnée Psychologique*, (2), 363–365.

Gebuis, T., & Reynvoet, B. (2011). Generating nonsymbolic number stimuli. *Behavior Research Methods*, *43*(4), 981–986.

Gilmore, C. K., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., … Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One*, *8*(6), e67374.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature*, *447*(7144), 589–591.

Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition*, *115* (3), 394–406.

Ginsburg, H. P., & Baroody, A. J. (1990). *Examiners manual of the test of early mathematics ability* (2nd ed). TX, Pro-Ed.

Girelli, L., Lucangeli, D. & Butterworth, B. (2000). The development of automaticity in accessing number magnitude. *Journal of Experimental Child Psychology*, 76, 104–122.

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive internet-based sample. *Proceedings of the National Academy of Sciences*, *109*(28), 11116–11120.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665–668.

Haverty, L. A., Koedinger, K. R., Klahr, D., & Alibali, M. W. (2000). Solving inductive reasoning problems in mathematics: Not-so-trivial pursuit. *Cognitive Science*, *24*(2), 249–298.

Hayslip, B., Maloy, R. M., & Kohl, R. (1995). Long-term efficacy of fluid ability interventions with older adults. *The Journals of Gerontology Series B: Psychological Sciences and Social Sciences*, *50B*(3), P141–P149.

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology*, *79*(2), 192–227.

Hedden, T., & Yoon, C. (2006). Individual differences in executive processing predict susceptibility to interference in verbal working memory. *Neuropsychology*, *20*(5), 511–528.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology*, *103*(1), 17–29.

Holzman, T. G., Pellegrino, J. W., & Glaser, R. (1983). Cognitive variables in series completion. *Journal of Educational Psychology*, *75*(4), 603–618.

Holzman, T. G., Pelligrino, J. W., & Glaser, R. (1982). Cognitive dimensions of numerical rule induction. *Journal of Educational Psychology*, *74*(3), 360–373.

Inglis, M., Attridge, N., Batchelor, S., & Gilmore, C. K. (2011). Non-verbal number acuity correlates with symbolic mathematics achievement: But only in children. *Psychonomic Bulletin & Review*, *18*(6), 1222–1229.

Iuculano, T., Tang, J., Hall, C. W., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science*, *11*(5), 669–680.

Jia, X., Liang, P., Lu, J., Yang, Y., Zhong, N., & Li, K. (2011). Common and dissociable neural correlates associated with component processes of inductive reasoning. *Neuroimage*, *56*(4), 2292–2299.

Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction*, *25*, 95–103.

Koponen, T., Aunola, K., Ahonen, T., & Nurmi, J.-E. (2007). Cognitive predictors of single-digit and procedural calculation skills and their covariation with Reading skill. *Journal of Experimental Child Psychology*, *97*(3), 220–241.

Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual–spatial working memory, and preschool quantity–number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, *103*(4), 516–531.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition*, *93*(2), 99–125.

Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of Experimental Child Psychology*, *103*(4), 546–565.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*(6), 1753–1767.

Liang, P., Zhong, N., Wu, J., Lu, S., Liu, J., & Yao, Y. (2007). *Time dissociative characteristics of numerical inductive reasoning: Behavioral and ERP evidence*. Paper presented at the Neural Networks, 2007. IJCNN 2007. International Joint Conference on.

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science*, *14*(6), 1292–1300.

Libertus, M. E., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability? *Learning and Individual Differences*, *25*, 126–133.

Libertus, M. E., Odic, D., & Halberda, J. (2012). Intuitive sense of number correlates with math scores on college-entrance examination. *Acta Psychologica*, *141*(3), 373–379.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition*, *121*(2), 256–261.

Massa, L. J., & Mayer, R. E. (2006). Testing the ATI hypothesis: Should multimedia instruction accommodate verbalizer-visualizer cognitive style? *Learning and Individual Differences*, *16*(4), 321–335.

Mayer, R. E., & Massa, L. J. (2003). Three facets of visual and verbal learners: cognitive ability, cognitive style, and learning preference. *Journal of Educational Psychology*, *95*(4), 833–846.

Mayer, R. E., Tajika, H., & Stanley, C. (1991). Mathematical problem solving in Japan and the United States: A controlled comparison. *Journal of Educational Psychology*, *83*(1), 69–72.

Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' precision of the approximate number system predicts later school mathematics performance. *PLoS One*, *6*(9), e23749.

Mundy, E., & Gilmore, C. K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology*, *103*(4), 490–502.

Mussolin, C., De Volder, A., Grandin, C., Schlögel, X., Nassogne, M.-C., & Noël, M.-P. (2010). Neural correlates of symbolic number comparison in developmental dyscalculia. *Journal of Cognitive Neuroscience*, *22*(5), 860–874.

Mussolin, C., Mejias, S., & Noël, M.-P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, *115*(1), 10–25.

Noël, M.-P., & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: An explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience*, *5*, 165.

Nuerk, H.-C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? putting the tens and units back in different bins. *Cognition*, *82*(1), B25–B33.

Nunes, T., Bryant, P., Barros, R., & Sylva, K. (2012). The relative importance of two different mathematical abilities to mathematical achievement. *British Journal of Educational Psychology*, *82*(1), 136–156.

Nunes, T., Campos, T. M., Magina, S., & Bryant, P. (2001). Introdução à Educação Matemática: os números e as operações numéricas. São Paulo: Proem.

Obersteiner, A., Reiss, K., & Ufer, S. (2013). How training on exact or approximate mental representations of number can enhance first-grade students' basic number processing and arithmetic skills. *Learning and Instruction*, *23*, 125–135.

de Oliveira Ferreira, F., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2012). Explaining school mathematics performance from symbolic and nonsymbolic magnitude processing: Similarities and differences between typical and low-achieving children. *Psychology & Neuroscience*, *5*(1), 37–4606.

Park, J., & Brannon, E. M. (2013). Training the approximate number system improves math proficiency. *Psychological Science*, *24*(10), 2013–2019. doi:10.1177/0956797613482944

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., … Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33–41.

Polya, G. (1957). Mathematics and plausible reasoning. volume I: Induction and analogy in mathematics; volume II: Patterns of plausible reasoning. JSTOR.

Praet, M., Titeca, D., Ceulemans, A., & Desoete, A. (2013). Language in the prediction of arithmetics in kindergarten and grade 1. *Learning and Individual Differences*, *27*, 90–96.

Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods*, *40*(3), 879–891.

Raven, J. (1998). *Court JH. Manual for Non-verbal matrix reasoning and vocabulary scales* (p. 12, G60p). Oxford: Oxford Psychologists.

Räsänen, P., Salminen, J., Wilson, A. J., Aunio, P., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. *Cognitive Development*, *24*(4), 450–472.

Redick, T. S., Unsworth, N., Kelly, A. J., & Engle, R. W. (2012). Faster, smarter? Working memory capacity and perceptual speed in relation to fluid intelligence. *Journal of Cognitive Psychology*, *24*(7), 844–854.

Rohde, T. E., & Thompson, L. A. (2007). Predicting academic achievement with cognitive ability. *Intelligence*, *35*(1), 83–92.

Rousselle, L., & Noel, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs. non-symbolic number magnitude processing. *Cognition*, *102*(3), 361–395.

Salthouse, T. A., & Mitchell, D. R. (1990). Effects of age and naturally occurring experience on spatial visualization performance. *Developmental Psychology*, *26*(5), 845–854.

Sasanguie, D., De Smedt, B., Defever, E., & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology*, *30*(2), 344–357.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology*, *114*(3), 418–431.

Sasanguie, D., Van den Bussche, E., & Reynvoet, B. (2012). Predictors for mathematics achievement? evidence from a longitudinal study. *Mind, Brain, and Education*, *6*(3), 119–128.

Shepard, R. N., & Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science*, *171*(3972), 701.

Smith, P., Fernandes, C., & Strand, S. (2001). *Cognitive abilities test 3: Technical manual and pupil book*. Windsor: NFER-Nelson.

Soltész, F., Szűcs, D., & Szűcs, L. (2010). Relationships between magnitude representation, counting and memory in 4-to 7-year-old children: A developmental study. *Behavioral and Brain Functions*, *6*(1), 1–14.

Swanson, L., & Kim, K. (2007). Working memory, short-term memory, and naming speed as predictors of children's mathematical performance. *Intelligence*, *35*(2), 151–168.

Tan, L. H., Liu, H.-L., Perfetti, C. A., Spinks, J. A., Fox, P. T. & Gao, J.-H. (2001). The neural system underlying Chinese logograph Reading. *Neuroimage*, 13, 836–846.

Tan, L. H., Spinks, J. A., Feng, C. M., Siok, W. T., Perfetti, C. A., Xiong, J., Fox, P. T. & Gao, J. H. (2003). Neural systems of second language Reading are shaped by native language. *Human Brain Mapping*, 18, 158–166.

Tudusciuc, O., & Nieder, A. (2007). Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex. *Proceedings of the National Academy of Sciences*, *104*(36), 14513–14518.

Turconi, E., Jemel, B., Rossion, B., & Seron, X. (2004). Electrophysiological evidence for differential processing of numerical quantity and order in humans. *Cognitive Brain Research*, *21*(1), 22–38.

Vanbinst, K., Ghesquière, P., & De Smedt, B. (2012). Numerical magnitude representations and individual differences in children's arithmetic strategy use. *Mind, Brain, and Education*, *6*(3), 129–136.

Vigneau, F., & Bors, D. (2001). *Visuospatial and verbal-analytic processes in Raven's Progressive Matrices performance*. Paper presented at the XI Meeting of the Canadian Society for Brain, Behaviour, and Cognitive Science (CSBBCS), Université Laval, Quebec City.

Vigneau, F., Caissie, A. F., & Bors, D. A. (2006). Eye-movement analysis demonstrates strategic influences on intelligence. *Intelligence*, *34*(3), 261–272.

Wechsler, D. (1992). *WISC-III UK manual*. Sidcup, Kent: The Psychological Corporation, Harcourt Brace Jovanovich.

Wechsler, D. (2001). *Manual for the Wechsler Test of Adult Reading (WTAR)*. San Antonio, TX: The Psychological Corporation.

Wei, W., Lu, H., Zhao, H., Chen, C., Dong, Q., & Zhou, X. (2012). Gender differences in children's arithmetic performance are accounted for by gender differences in language abilities. *Psychological Science*. doi:0956797611427168

Wei, W., Yuan, H., Chen, C., & Zhou, X. (2012). Cognitive correlates of performance in advanced mathematics. *British Journal of Educational Psychology*, *82*(1), 157–181.

Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of" The number race," an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions*, *2*(1), 1–16.

Woodcock, R., McGrew, K., & Mather, N. (2001). Woodock johnson III test of achievement. *Rolling Meadows, IL: Riverside Publishing*.

Yu, X., Liu, J., Li, D., Liu, H., Cui, J., & Zhou, X. (2015). Dynamic mental number line in simple arithmetic. *Psychological Research*, *80*(3), 410–421.

Zhou, F., Zhao, Q., Chen, C., & Zhou, X. (2012). Mental representations of arithmetic facts: evidence from eye movement recordings supports the preferred operand-order-specific representation hypothesis. *The Quarterly Journal of Experimental Psychology*, *65*(4), 661–674.

Zhou, X., & Cheng, D. (2015). When and why numerosity processing is associated with developmental dyscalculia. In S. Chinn (Ed.), The routledge international handbook of dyscalculia and mathematical learning difficulties (pp. 78–89). New York: Routledge.

Zhou, X., Wei, W., Zhang, Y., Cui, J., & Chen, C. (2015). Visual perception can account for the close relation between numerosity processing and computational fluency. *Frontiers in Psychology*, *6*, 1364.

Appendix. Twelve arithmetical rules in number series completion

	Rule in number series completion	Example
1	a, a + 1, a + 2, a + 3, a + 4: (a + 5)	3, 4, 5, 6, 7: (8)
2	a, a–1, a–2, a–3, a–4: (a–5)	9, 8, 7, 6, 5: (4)
3	a, a + 1, a + 1 + 3, a + 1 + 3 + 5, a + 1 + 3 + 5 + 7: (a + 1 + 3 + 5 + 7 + 9)	3, 4, 7, 12, 19: (28)
4	a, a–2, a–2–3, a–2–3–4, a–2–3–4–5: (a–2–3–4–5–6)	22, 20, 17, 13, 8: (2)
5	a, a + 1, a + 2, a + 3, a + 2, a + 1: (a)	1, 2, 3, 4, 3, 2: (1)
6	a, a–1, a–1–3, a–1–3–5, a–1–3: (a–1)	16, 15, 12, 7, 12: (15)
7	a, a + 1, 2a + 1, 3a + 2, 5a + 3: (8a + 5)	1, 2, 3, 5, 8: (13)
8	a, a + 3, a + 3 + 4, a + 3 + 4–6, a + 3 + 4–6 + 3, a + 3 + 4–6 + 3 + 4: (a + 3 + 4–6 + 3 + 4–6)	2, 5, 9, 3, 6, 10: (4)
9	a, a × 2, a × 2 × 2, a × 2 × 2 × 2, a × 2 × 2 × 2 × 2: (a × 2 × 2 × 2 × 2 × 2)	2, 4, 8, 16, 32: (64)
10	a, a × 1, a × 1 × 2, a × 1 × 2 × 3, a × 1 × 2 × 3 × 4: (a × 1 × 2 × 3 × 4 × 5)	2, 2, 4, 12, 48: (240)
11	a, a × 3, a × 3 ÷ 2, a × 3 ÷ 2 × 3, a × 3 ÷ 2 × 3 ÷ 2: (a × 3 ÷ 2 × 3 ÷ 2 × 3)	8, 24, 12, 36, 18: (54)
12	a, b, a + 2, b × 3, a + 2 + 2, b × 3 × 3: (a + 2 + 2 + 2)	2, 3, 4, 9, 6, 27: (8)