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CASE REPORT

Modality-Dependent or Modality-Independent Processing in Mental Arithmetic: Evidence From Unimpaired Auditory Multiplication for a Patient With Left Frontotemporal Stroke

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Abstract

Objectives: Mental arithmetic is essential to daily life. Researchers have explored the mechanisms that underlie mental arithmetic. Whether mental arithmetic fact retrieval is dependent on surface modality or knowledge format is still highly debated. Chinese individuals typically use a procedure strategy for addition; and they typically use a rote verbal strategy for multiplication. This provides a way to examine the effect of surface modality on different arithmetic operations. **Methods:** We used a series of neuropsychological tests (i.e., general cognitive, language processing, numerical processing, addition, and multiplication in visual and auditory conditions) for a patient who had experienced a left frontotemporal stroke. **Results:** The patient had language production impairment; but preserved verbal processing concerning basic numerical abilities. Moreover, the patient had preserved multiplication in the auditory presentation rather than in the visual presentation. The patient suffered from impairments in an addition task, regardless of visual or auditory presentation. **Conclusions:** The findings suggest that mental multiplication could be characterized as a form of modality-dependent processing, which was accessed through auditory input. The learning strategy of multiplication table recitation could shape the verbal memory of multiplication leading to persistence of the auditory module. (*JINS*, 2017, *23*, 692–699)

Keywords: Left hemisphere, Language, Mathematics, Multiplication, Neuropsychological tests, Stroke

INTRODUCTION

People rely heavily on arithmetic to process numbers and solve simple calculation problems in daily life. Many studies have investigated how the human brain processes simple arithmetic (Butterworth, Zorzi, Gierlli, & Jonckheere, 2001; Campbell & Epp, 2004; Campbell & Metcalfe, 2008; Dehaene & Cohen, 1995, 1997). For example, Campbell and Epp (2004) suggested that solving a simple arithmetic problem (e.g., $8+5$) involves three stages: encoding the arithmetic problem, retrieving or calculating the answer, and reporting the answer. Therefore, there are at least two ways to solve arithmetic problems: the retrieval of arithmetic facts and calculating quantity. Therein, arithmetic facts knowledge, such as memory retrieval of basic table facts (e.g., $2+4 = 6$; $3\times4 = 12$), is essential (Megías & Macizo, 2015). The modality or format of arithmetic presentations could affect the encoding of problem and reporting of answers during arithmetic (Dehaene & Cohen, 1995; McCloskey, 1992). However, whether retrieval of arithmetic facts or substantial computation depends on the modality of external input is controversial.

According to the modality-dependent processing viewpoint, arithmetic fact retrieval is dependent on the modality of presentation (e.g., visual or auditory). From this viewpoint, multiple-code models (e.g., encoding-complex model) indicate that the modality of presentation is encoded to create multimodal concepts with a specific format (Campbell & Clark, 1988; Campbell, 1994). Supportive evidence showed

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that patients with left ventral temporo-occipital infarctions are impaired in addition calculations, but only when the operands are presented visually, not when they are presented auditorily (Cohen & Dehaene, 1995; McNeil & Warrington, 1994). The pure alexic patient could not verbally produce the correct answers in multiplication tasks, demonstrating that multiplication processing was affected by verbally reading (Cohen & Dehaene, 2000).

According to the modality-independent processing viewpoint, surface modality does not affect mental arithmetic fact retrieval. Specifically, the abstract code model proposes that arithmetic facts are addressed and retrieved by the comprehension system into abstract codes, which are then used for the retrieval of arithmetic facts or for the calculation of an answer (McCloskey, 1992; McCloskey & Macaruso, 1995). For the application, arithmetic operations are represented in symbolic formats abstracted from their surface form (e.g., visual or auditory). Consistent with this view, brain-injured patients show specific deficits regardless of presentation format. For both written and oral presentation and response modes, patient M.W.'s performance pattern consistently showed difficulty in retrieving multiplication facts (McCloskey, Carmazza, & Basili, 1985).

The inconsistent results concerning arithmetic (e.g., addition and multiplication) may be due to different arithmetic learning experiences and strategies used. That is, schoolchildren are usually taught to use procedural strategies for simple addition and subtraction; however, they are taught to use verbal memory strategy to memorize multiplication facts (Dehaene & Cohen, 1997; Roussel, Fayol, & Baarrouillet, 2002; Zhou & Dong, 2003). These differential strategies during the acquisition of arithmetic facts may play an important role in shaping their mental representations (Siegler & Shipley, 1995).

Interestingly, Chinese individuals typically use special learning strategies for mental arithmetic that are different from Western countries. Previous studies showed that a substantial number of American and Canadian adults (up to 29%) solve single-digit addition and multiplication problems *via* procedural strategies (Campbell & Xue, 2001; Lefevre & Liu, 1997). In China, the differential use of these two strategies for different arithmetic operations is especially clear. For addition and subtraction, varieties of procedures (e.g., transforming $4+7=3+7+1=11$) are used; however, for multiplication, Chinese children are instructed to exclusively use the rote verbal strategy (Zhou & Dong, 2003).

Generally, Chinese children start to memorize the multiplication table by rote verbal strategy during the second semester of the first grade or the first semester of the second grade. Since then, they use the strategy of multiplication table recitation to resolve multiplication problem (Zhou, Chen, Zang, et al., 2007). The learning experience could shape memory of arithmetic facts. Previous studies have shown that the multiplication is typically represented with verbal code, but the procedure-based arithmetic is typically represented with visuospatial code (e.g., Dehaene & Cohen, 1995; Zhou, Chen, Zang, et al., 2007). The auditory presenting of arithmetic problems might have more advantage to the multiplication problems due to the verbal memory. Therefore, Chinese participants' acquisition strategy preferences provide a well-testable model to examine the effect of perceptual modality on different arithmetic operations.

We investigated whether mental arithmetic retrieval depends on surface modality and whether there is a dissociation between addition and multiplication in a stimulus-dependent manner. Although a previous study demonstrated the influence of surface form on addition calculation depending on the type of the input stimulation (i.e., Arabic digit, number word, and dot notation) (Sciama, Semenza, & Butterworth, 1999), how the surface modality or knowledge format (i.e., visual and auditory presentation) affects mental arithmetic is still unclear.

This study reports examined one patient (Y.Q.) with 11 years of formal education in China. When he was 45 years old, he suffered a cerebral infarction in the left frontotemporal lobe and basal ganglia. Before the assessments of cognitive abilities for the current investigation, clinical neuropsychological examination showed that the patient had prevalent loss of arithmetic [zero score in Mini Mental State Examination (MMSE)]. His family reported that he could understand other persons' spoken language. To comprehensively examine the patient's cognitive ability, we investigated general cognitive, language, and numerical processing. Previous studies suggested that mental addition and multiplication depended on differential learning and practice experiences.

For example, multiplication facts typically are based on verbal-motor areas, including supplementary motor area, precentral gyrus (i.e., primary motor area) in Chinese adults (Zhou, Chen, Zang, et al., 2007). Meanwhile, the verbal memory of multiplication facts is highly modularized. For example, Mainland Chinese participants memorized only the smaller-operand-first multiplication problems (Zhou, Chen, Zhang, et al., 2007). So, we used a typical auditory input for multiplication, which is maybe more targeted to the verbal memory of multiplication. We expected that multiplication processing would be more affected by the auditory form.

METHODS

Ethical Consideration

The ethics board of Beijing Normal University approved the study protocol and all participants provided written informed consent.

PARTICIPANTS

Patient

Y.Q. was a 45-year-old right-handed man with 11 years of education, who previously worked in the transportation department of the Chinese Railways. In January 2012, his right limbs showed movement disability without obvious cause, and he talked in a confused and irrational manner.

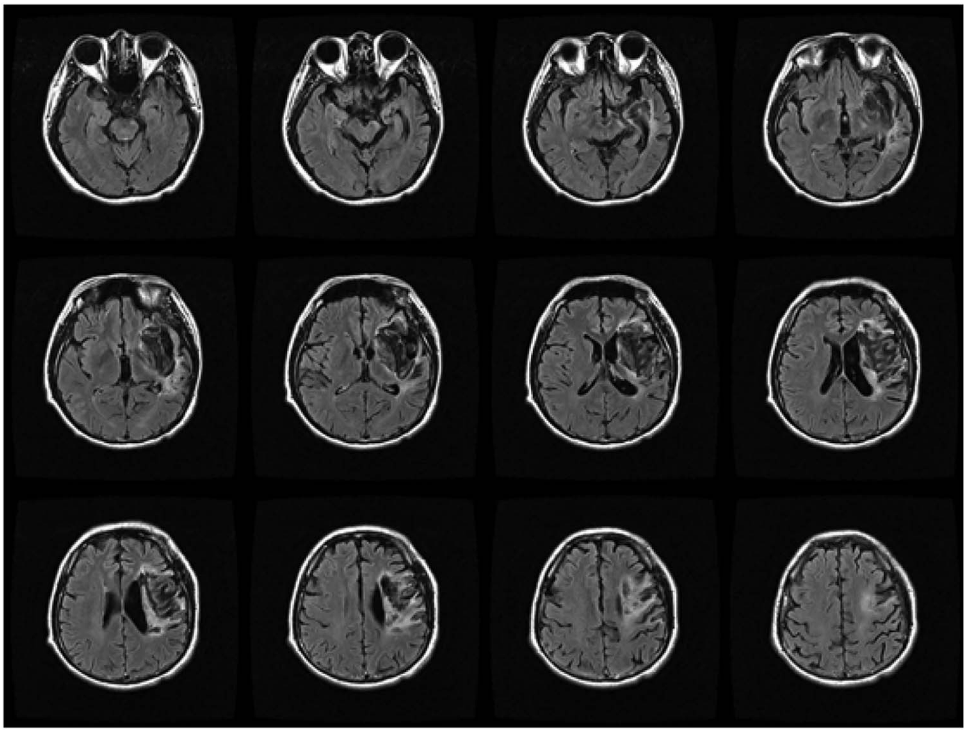

Fig. 1. T2-weighted MRI images for patient Y.Q. The right side of each image refers to the left hemisphere. The patient had severe lesions in the left frontotemporal lobe and basal ganglia due to cerebral infarction. No obvious lesions were found in the parietal lobes.

When he walked, his right leg sloped to the right. The next morning, he displayed hyperspasmia once and could not recognize his family. Magnetic resonance imaging (MRI) examination of the brain indicated that he had an arteriosclerotic cerebral infarction in the left frontotemporal lobe and basal ganglia, but no lesions in the parietal lobes (Figure 1). The patient exhibited incoherent consciousness disorder lasting for 1 month. Then, he regained coherent consciousness; however, he could not distinguish between his own right and left finger. In daily life, the patient could not go shopping independently and complained about calculation difficulties. However, he could understand other persons' spoken language. In addition, he had a 3-year medical history of hypertension, hyperlipemia, and fatty liver hepatitis. According to the MMSE, the patient was impaired in attention, calculation, memory, and language (all $ps < .05$); however, he showed normal scores for orientation and expression on the MMSE (Folstein, Folstein, & McHugh, 1975) (Table 1).

Table 1. Mini-Mental State Examination in patient Y.Q. and controls

Test name	Y.Q.	Controls
Orientation	10/10	10/10
Expression	3/3	3/3
Attention and calculation	0/5	5/5
Memory	1/3	3/3
Language	3/9	9/9

Note. When the number of years of education >6, a cutoff of 24 was chosen to indicate normal cognitive function.

Controls

Control participants were six healthy volunteers (three men, three women) who were matched with the patient in age and education years. Exclusion criteria included a history of neurological disorder, head injury, or alcohol abuse. Their mean age was 46.3 years (SD = 1.6), and they had an average of 10.7 years (range from 10 to 11 years; SD = 0.52) of education. There were no significant differences between controls and the patient in age and education years ($ps > .48$). The ethics board of Beijing Normal University approved the study protocol and each of the participants provided written informed consent.

Assessments of Cognitive Abilities

General cognitive tests, language processing tests, and numerical processing tests were used to assess cognitive abilities. To examine the influence of presentation modality of arithmetic problem on mental arithmetic, the experiments of addition and multiplication were designed. All the tasks were programmed using Web-based applications available at www.dweipsy.com/lattice (Wei et al., 2012). The first completed responses were scored in each test when the participants offered several responses (see the Supplementary Material for details of cognitive tests).

RESULTS

For all neuropsychological tests, the participants' first complete responses were scored, and the correct scores were analyzed. To examine the potential differences in performance between patient Y.Q. and controls across all tests, we used a χ^2 test, a modified *t* test, and a program developed by Crawford, Garthwaite, and Porter (2010), which refers to the patient's scores relative to a control sample. The modified *t* test also provides a point estimate of the effect size for the difference between the case and controls (z-cc) with an accompanying 95% confidence interval. For smaller-operand-first and larger-operand-first problems, a χ^2 test was used to examine the patient's performance of addition and multiplication in the visual and auditory condition.

General Cognitive Processing

The scores of general cognitive processing for the patient and controls are displayed in Table 2. To compare the scores, *t* tests were used. For Raven's Progressive Matrices, mental rotation, forward digit span, backward digit span, and choice reaction time, the differences between controls and patient Y.Q. were not significant (all $ps > .1$).

Language Processing

The language processing scores for the patient and controls are displayed in Table 3. To compare the scores, *t* tests were used. For word repetition, the patient showed lower scores than did the controls, $t(5) = 3.87$, $p < .05$, z-cc = −4.18 (−6.80 to −1.55). Patient Y.Q. showed significantly lower performance than did controls in Chinese character reading, $t(5) = 11.04$, $p < .01$, z-cc = −11.89 (−19.08 to −4.79); picture naming, $t(5) = 2.65$, $p < .05$, z-cc = −2.90 (−4.80 to −0.97); verbal fluency, $t(5) = 4.96$, $p < .01$, z-cc = −5.36 (−8.67 to −2.07); and word rhyming, $t(5) = 6.01$, $p < .01$, z-cc = −6.49 (−10.46 to −2.55). There were no significant differences between patient Y.Q. and controls in sentence completion ($ps > .05$).

Numerical Processing

The scores of numerical processing for the patient and controls are displayed in Table 4. Because the control group achieved full scores in numerical processing except for the numerical magnitude comparison task, the *t* test precondition was not met. Therefore, χ^2 tests were used to compare the scores. For dot counting, abstract counting, 1-digit and 2-digit Arabic numeral reading, 1-digit Chinese number word reading, 1-digit and 2-digit Arabic number dictating, numerical magnitude comparison, comparison of two dot arrays, and number series completion, there were no significant differences between patient Y.Q. and controls (all $ps > .05$). The patient had lower scores than did the controls for backward verbal number sequence, $\chi^2(1) = 5.55$, $p < .05$; 3-digit Arabic numeral reading, $\chi^2(1) = 16.64$, $p < .001$; 4-digit Arabic numeral reading, $\chi^2(1) = 16.64$, $p < .001$; 2-digit Chinese number word reading, $\chi^2(1) = 5.17$, $p < .05$; simple division, $\chi^2(1) = 26.59$, $p < .001$; simple addition, $t(5) = 11.65$, $p < .001$, z-cc = −12.58 (−20.19 to −5.08); simple subtraction, $t(5) = 5.91$, $p < .01$, z-cc = −6.38 (−10.29 to −2.50); and simple multiplication, $t(5) = 12.41$, $p < .001$, z-cc = −13.40 (−21.51 to −5.42).

Table 2. Scores on the general cognitive tasks in patient Y.Q. and controls

Test name	Y.Q.	Controls
Raven's Progressive Matrices	11/15	13.7/15
Mental rotation	8/10	8.3/10
Forward digit span	5	6.83
Backward digit span	3	5.67
Choice reaction	30/30	30/30

Note. The numerator refers to the number of correct trials; the denominator refers to the total number of trials for each task.

Table 3. Scores on the language processing tasks

Test name	Y.Q.	Controls
Word repetition	5/10	9.3/10
Chinese character reading	20/36	34.5/36
Picture naming	47/60	55/60
Verbal fluency	2.75	12.83
Word rhyming	7/40	34.7/40
Sentence completion	17/20	17.2/20

Note. The numerator refers to the number of correct trials; the denominator refers to the total number of trials for each task.

Addition and Multiplication in Visual and Auditory Condition

The patient's performance in addition and multiplication of the visual and auditory condition is shown in Figure 2 and

Table 4. Scores on the numerical processing tasks

Test name	Y.Q.	Controls
Dot counting	9/10	10/10
Forward verbal number sequence	2/2	2/2
Backward verbal number sequence	0/2	2/2
Arabic numeral reading (1-digit)	9/10	10/10
Arabic numeral reading (2-digit)	5/6	6/6
Arabic numeral reading (3-digit)	0/6	6/6
Arabic numeral reading (4-digit)	0/6	6/6
Chinese number word reading (1-digit)	9/10	10/10
Chinese number word reading (2-digit)	3/6	6/6
Arabic number dictating (1-digit)	8/10	10/10
Arabic number dictating (2-digit)	4/6	6/6
Number comparison (single digit)	57/60	57.8/60
Comparison of two dot arrays	65.17	63
Simple addition	8	47
Simple subtraction	10	47.33
Simple multiplication	10	55.17
Simple division	4/20	20/20
Number series completion	7	12.33

Note. The numerator refers to the number of correct trials; the denominator refers to the total number of trials for each task.

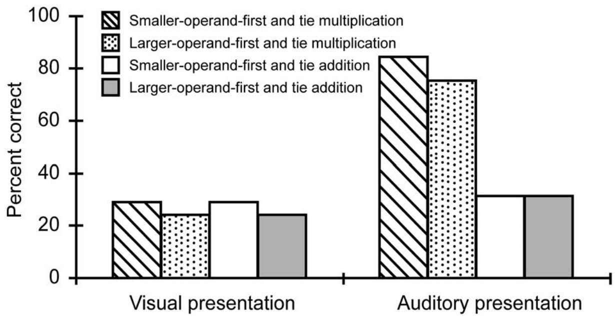

Fig. 2. The patient's percentage correct responses for single-digit multiplication and addition problems. The problems included smaller-operand-first (e.g., 3 + 7, 3 × 7), larger-operand-first (e.g., 7 + 3, 7 × 3), and tie problems.

Appendices 1 and 2. For smaller-operand-first and tie addition and larger-operand-first and tie addition, there were no significant differences between visual and auditory conditions, $\chi^2(1) = 0.05$, $p = .82$; $\chi^2(1) = 0.55$, $p = .46$, respectively. For smaller-operand-first and tie multiplication and larger-operand-first and tie multiplication, patient Y.Q. showed higher scores in the auditory than visual condition, $\chi^2(1) = 28.28, p < .001$; $\chi^2(1) = 35.50, p < .001$, respectively.

DISCUSSION

The present study aimed to investigate the influence of presentation modality of arithmetic problem on mental arithmetic for a patient with infarction of the left anterior hemisphere (i.e., left frontotemporal lobe and basal ganglia). The results showed that the patient was impaired in the addition task, regardless of visual or auditory presentation; however, he showed preserved multiplication in the auditory condition. This suggests that mental multiplication could be characterized as modality-dependent processing, which was accessed through auditory input.

Compared to controls, Y.Q. had generally impaired abilities in the numerical and arithmetic tasks. Although parietal lobes contribute to simple calculation (multiplication or addition) (Della Puppa et al., 2013; Semenza, Salillas, De Pallegrin, & Della Puppa, 2016), the frontal cortex is also closely related to the demands imposed by arithmetic fact retrieval (Jost, Khader, Burke, Bien, & Rösler, 2011). Neuropsychological studies have suggested that anterior parts of the left hemisphere including the frontal cortex (Fasotti, Elilng, & Bremer, 1992; Lucchelli & De Renzi, 1993; Tohgi et al., 1995) and basal ganglia (Dehaene & Cohen, 1997) play an important role in mathematical and language processing.

First, patients with frontal lobe lesion show deficits in numerical processing (Cappelletti, Butterworth, & Kopelman, 2012; Fasotti et al., 1992; Lucchelli & De Renzi, 1993). For example, five frontal lesion patients showed impaired performance in number reading, single-digit arithmetical operations (i.e., addition, subtraction, multiplication, and division problems), multi-digit calculation, and reading (Cappelletti et al., 2012). Furthermore, some researchers have also suggested that the frontal lobes are involved in two components of the calculation system: retrieval of basic arithmetical facts and execution of calculation procedures (Sokol, McCloskey, Cohen, and Aliminosa, 1991).

Second, deficits in arithmetic have been reported after left subcortical lesions that affect the basal ganglia (Benke, Delazer, Bartha, & Auer, 2003; Dehaene & Cohen, 1997; Delazer et al., 2004; Hittmair-Delazer, Semenza, & Denes, 1994). Two patients affected by basal ganglia lesions showed impairment in a timed mental arithmetic task and writing complex problems (Benke et al., 2003). It has also been suggested that cortico-subcortical loops involving the basal ganglia are part of a functional circuit mediating language-based retrieval of simple addition and multiplication facts (Dehaene & Cohen, 1997).

Building on the extensive evidence on the role of the left anterior hemisphere in most numerical and language abilities, the relatively preserved auditory multiplication ability in Y.Q. was rather interesting. We infer that the left frontotemporal lobe and basal ganglia lesions impaired the ability to execute the calculation procedure that was important to learn addition facts (Sokol et al., 1991); however, the intact linguistic motor areas sustained the verbal memory for the multiplication table (Zhou, Chen, Zang, et al., 2007). Studies have demonstrated that schoolchildren are taught to use procedural strategies for simple addition such as counting and transformation (e.g., $6 + 7 = 6 + 6 + 1$, $9 + 7 = 9 + 1 + 6$), while they are taught to use a verbal memory strategy to memorize multiplication facts (Dehaene & Cohen, 1997; Roussel et al., 2002; Zhou, Chen, Zang, et al., 2007).

With the verbal memory strategy, people repeatedly recite multiplication tables that could be stored as verbal memory. Moreover, functional MRI studies showed that multiplication activated more the precentral gyrus, supplementary motor areas, and posterior and anterior superior temporal gyrus relative to addition (Lee, 2000; Zhou, Chen, Zang, et al., 2007). This confirmed that learning multiplication relies heavily on verbal memory and is related to linguistic motor areas.

There was the dissociation between addition and multiplication in auditory modality but not in visual modality. Previous studies have shown lots of dissociations among simple arithmetic. The main finding is the dissociation between procedure-based arithmetic (e.g., addition) and verbal-based arithmetic (e.g., multiplication). The patients with verbal processing deficits typically had problems in multiplication but not in the procedure-based arithmetic (e.g., Cohen & Dehaene, 2000; Dehaene & Cohen, 1997; Delazer & Benke, 1997; Hittmair-Delazer et al., 1994; Puvanendran, Dowker, & Demeyere, 2016). For example, Broca's aphasia had preserved performance in simple addition, but can retrieve no multiplication facts (Hittmair-Delazer et al., 1994). The pure alexic patient, although unable to read correctly the operands of visually presented problems, could still produce verbally the exact result of the very same problems in subtraction, addition, and division tasks rather than multiplication task (Cohen & Dehaene, 2000). To the contrary, the patients with normal verbal processing could have regular multiplication performance but have impaired performance in the procedure-based arithmetic. For example, a Gerstmann's syndrome apraxia patient cannot do simple addition and subtraction, but had preserved language and multiplication abilities (Delazer & Benke, 1997).

Furthermore, Cohen and Dehaene (2000) also demonstrate a dissociation between multiplication and addition in auditory and visual presentation. The pattern was that the alexic patient impaired only in multiplication on visual Arabic input, but showed preserved performance in addition, subtraction, multiplication and division on orally input. The auditory input in the study could be transformed into visual code for addition, subtraction, and division, which were intact in visual modality. But the multiplication processing was more associated with verbally reading that could not realize the transformation. Therefore, the pattern of dissociation is different from the present results. In the present study, we found the dissociation between addition and multiplication in auditory modality but not in visual modality. We infer that the difference may be because the differential learning or practice strategies between Chinese and Western countries. Chinese individuals typically use the rote verbal strategy for multiplication. Auditory input could easily activate the verbal code to improve problem solving.

In the present study, the patient had a relatively preserved multiplication ability in auditory presentation. The auditory input format directly induced this verbal memory strategy for multiplication. Besides the contributing factor of rote verbal strategy, there may be other factor contributed to the pattern of dissociation. Even though the patient was impaired in language production and most numerical processing abilities compared to controls, he showed preserved verbal processing in basic numerical abilities (e.g., counting, Arabic number reading, number word reading, and Arabic number dictating) and semantic memory.

One of the factors might be the patient's prevalent loss of simple arithmetic in Arabic digit input. The injuries in frontotemporal lobe and basal ganglia led a prevalent loss of simple arithmetic in Arabic digit input. The loss might be a type of production deficit, which might be equivalent to his overall language production deficits (including word rhyming, Chinese character reading, word repetition, verbal fluency, and naming of characters with low frequency). The auditory input could be helpful for the retrieval of the verbal-based multiplication because the auditory input could directly activate the verbal code and thus mitigate the production load.

However, the poor performance of addition in the visual modality could not be improved in auditory modality. For example, the patient in Cohen and Dehaene's study (2000) had nearly perfect performance in addition, subtraction and division on visual input, and also had similar performance on auditory input. It indicated that performances of addition, subtraction and division in visual modality were not affected by auditory modality. Therefore, the current findings support the notion of modality-dependent processing in multiplication. The learning strategy of multiplication table recitation could shape the verbal memory of multiplication.

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Supplementary material

To view supplementary material for this article, please visit https://doi.org/10.1017/S1355617717000479

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APPENDIX 1

Patient Y.Q.'s Single-Digit Multiplication Performance for Visual and Auditory Presentation

		Second operand																	
		1		2		3		4		5		6		7		8		9	
		V	A	V	A	V	A	V	A	V	A	V	A	V	A	V	A	V	A
First operand	1	1	1	2	1	8	3	8	4	5	5	8	6	8	7	8	8	8	9
	2	2	2	4	4	24	6	27	18	12	10	16	12	16	18	16	16	27	18
	3	3	3	36	4	27	9	36	27	30	21	30	27	27	21	30	27	24	27
	4	4	4	18	8	27	12	36	16	30	20	54	12	16	42	36	32	48	36
	5	5	5	36	15	25	15	36	20	25	25	30	30	40	35	40	40	30	45
	6	8	6	18	12	27	18	36	24	45	30	36	36	38	42	40	48	27	54
	7	9	7	18	14	36	21	27	28	45	35	36	36	49	49	36	56	56	63
	8	8	8	18	16	27	25	36	32	36	40	72	48	35	32	81	64	40	16
	9	9	9	39	18	27	27	36	36	54	45	48	54	54	36	36	27	49	81

Note. The answers for problems of the range from 1 × 1 to 9 × 9 are demonstrated in the table. Shaded answers are incorrect. For all problems, the answers in the first row refer to the visual condition (V), whereas the answers in the second row refer to the auditory condition (A).

APPENDIX 2

Patient Y.Q.'s Single-Digit Addition Performance for Visual and Auditory Presentation

		Second operand																	
		1		2		3		4		5		6		7		8		9	
		V	A	V	A	V	A	V	A	V	A	V	A	V	A	V	A	V	A
First operand	1	2	2	3	3	4	2	6	5	6	2	12	7	6	5	5	1	6	6
	2	3	3	5	4	5	4	7	6	5	11	6	5	9	9	9	10	11	6
	3	4	4	3	3	8	4	5	7	5	7	11	10	11	5	11	12	11	6
	4	6	5	6	6	15	4	5	5	12	8	12	11	12	9	11	11	15	15
	5	6	6	7	6	9	7	7	11	12	12	12	11	15	7	15	11	12	10
	6	6	9	6	15	8	7	8	8	11	11	12	12	11	15	11	14	11	12
	7	3	8	11	8	8	5	15	15	12	7	12	5	12	14	15	10	13	15
	8	9	12	12	10	12	6	11	6	11	15	15	9	14	15	11	15	15	12
	9	10	10	11	10	11	11	11	12	12	11	15	12	11	12	15	12	14	14

Note. Answers for problems of the range from 1 + 1 to 9 + 9 are demonstrated. Shaded answers are incorrect. For all problems, answers in the first row refer to the visual condition (V) and answers in the second row to the auditory condition (A).