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The semantic system is involved in mathematical problem solving

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ABSTRACT

Numerous studies have shown that the brain regions around bilateral intraparietal cortex are critical for number processing and arithmetical computation. However, the neural circuits for more advanced mathematics such as mathematical problem solving (with little routine arithmetical computation) remain unclear. Using functional magnetic resonance imaging (fMRI), this study (N = 24 undergraduate students) compared neural bases of mathematical problem solving (i.e., number series completion, mathematical word problem solving, and geometric problem solving) and arithmetical computation. Direct subject- and item-wise comparisons revealed that mathematical problem solving typically had greater activation than arithmetical computation in all 7 regions of the semantic system (which was based on a meta-analysis of 120 functional neuroimaging studies on semantic processing). Arithmetical computation typically had greater activation in the supplementary motor area and left precentral gyrus. The results suggest that the semantic system in the brain supports mathematical problem solving.

Introduction

There are two distinctive components of mathematical abilities: arithmetical computation and mathematical problem solving (Fuchs et al., 2008b; Geary et al., 2000; Nunes et al., 2009, 2012; Wei et al., 2012b; Zhang et al., 2016). They involve differential cognitive mechanisms (Fuchs et al., 2008a; Nunes et al., 2012; Zhang et al., 2016). Over the last two decades, neuroimaging and neuropsychological studies have dramatically increased our understanding of the neural bases of arithmetical computation, but not mathematical problem solving. The current study used functional magnetic resonance imaging to examine neural basis of mathematical problem solving.

The dissociation between arithmetical computation and mathematical problem solving in developmental studies

Arithmetical computation refers to either direct retrieval of answers based on arithmetic facts or the use of computational procedures on digits (Fuchs et al., 2008a, 2008b). Mathematical problem solving, on the other hand, is a type of problem solving that focuses on the search of a solution path from problem preconditions to problem goal by trial-and-error. It is involves many mathematical tasks, but especially in mathematical word problem solving (e.g., Fuchs et al., 2008a, 2008b, 2014; Hickendorff, 2013; Wei et al., 2012b), number series completion (e.g., Inglis et al., 2011; Nunes et al., 2012; Wei et al., 2012b; Woodcock et al., 2001; Zhang et al., 2016), and geometric proofing (e.g., Epelboim and Suppes, 2001; Giofrè et al., 2013).

Previous developmental studies have shown a dissociation between arithmetical computation and mathematical problem solving (Fuchs et al., 2008a; Hickendorff, 2013; Nunes et al., 2012; Wei et al., 2012b; Zhang et al., 2016). First, there is at best a weak correlation between arithmetical computation and mathematical problem solving. For example, Wei et al. (2012b) found that arithmetical computation and mathematical problem solving (as measured by number series completion) were not significantly correlated. Moreover, both arithmetical computation and problem solving make independent contributions to mathematical achievement (e.g., Nunes et al., 2012; Powell and Fuchs, 2014).

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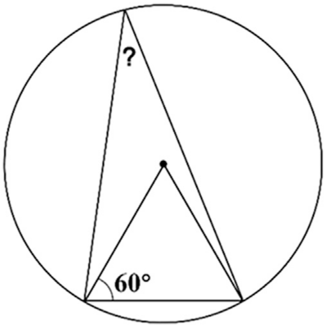

Fig. 1. A sample geometric problem (left side) and a sample arithmetic computation in the geometric problem context (right side).

Table 1
Differences in brain activation between mathematical problem solving and arithmetical computation: Subject-wise analysis.

Brain region	Volume	T value	Coordinates		
Mathematical problem solving – arithmetical computation					
Left middle frontal gyrus	3244	9.39	−30	18	48
		8.64	−36	21	54
		8.47	−21	30	57
Left middle occipital gyrus	3379	8.20	−42	−72	33
		8.16	−45	−54	39
		7.71	−45	−66	27
Right middle frontal gyrus (Orbital)	237	6.46	45	51	−12
		5.44	48	30	−6
		4.95	42	60	0
Right cerebellum (Crus2)	261	5.81	15	−84	−30
		5.20	33	−75	−36
		5.14	42	−72	−33
Right middle temporal gyrus	230	5.38	63	−45	−3
		4.70	63	−30	−12
		4.30	51	−51	−6
Right superior frontal gyrus	30	4.55	21	63	12
Left cerebellum (Crus2)	74	4.45	−33	−78	−33
		4.22	−15	−72	−30
		4.13	−27	−69	−36
Left caudate	17	4.14	−15	6	15
Left middle occipital gyrus	12	3.80	−27	−84	12
Left fusiform gyrus	13	3.59	−30	−42	−15
Arithmetical computation – mathematical problem solving					
Left middle cingulum	33	5.37	−12	15	33
Left supplementary motor area	38	4.33	−9	−3	57
Right supplementary motor area	11	4.30	12	6	48
Left precuneus	39	4.26	−24	−51	12
		4.19	−18	−45	9
		3.74	−33	−42	3
Left precentral gyrus	21	4.22	−48	−6	54
Left insula	16	3.94	−33	−21	30
Right precuneus	14	3.81	30	−45	3
Left middle cingulum	11	3.58	−24	−6	33

Note: Clusters with $p < 0.001$ (uncorrected) and a minimum size of 10 were considered statistically significant. Coordinates (XYZ) are in MNI space.

Table 2
Differences in brain activation between mathematical problem solving and arithmetical computation: Item-wise analysis.

Brain region	Volume	T value	Coordinates		
Mathematical problem solving – arithmetical computation					
Left angular gyrus	4441	11.61	−42	−66	48
		11.05	−48	−63	33
		10.96	−36	−75	45
Left superior frontal gyrus	3380	10.43	−21	33	57
		9.58	−39	12	57
		9.19	−36	21	54
Left cerebellum (Crus2)	140	7.08	−21	−84	−42
		6.60	−33	−84	−36
		4.47	−12	−81	−45
Right cerebellum (Crus2)	200	6.98	42	−72	−39
		6.69	15	−87	−30
		6.18	33	−81	−36
Right inferior frontal gyrus (Orbital)	162	5.81	51	45	−9
		5.78	42	57	−6
		5.43	45	33	−9
Left cerebellum (9)	67	5.06	−3	−57	−48
		4.14	−9	−51	−48
		3.74	12	−48	−45
Right parahippocampal gyrus	66	5.06	33	−33	−15
Right precuneus	20	4.38	24	−54	18
Left fusiform gyrus	50	4.23	−30	−45	−12
Left inferior frontal gyrus (Triangle)	21	3.87	54	36	15
		3.54	57	27	15
Right postcentral gyrus	30	3.84	48	−6	30
		3.69	48	−12	39
Right inferior frontal gyrus (Operculum)	10	3.72	57	15	18
Arithmetical computation – Mathematical problem solving					
Left lingual	58	5.75	−30	−54	3
		3.86	−21	−78	6
		3.54	−27	−66	6
Right supplementary motor area	196	5.58	9	9	48
		4.75	−12	−3	60
		3.94	−3	0	57
Right calcarine	41	5.26	30	−51	3
Left insula	265	5.19	−30	15	12
		5.12	−36	9	9
		4.86	−12	18	33
Left precentral gyrus	18	5.04	−48	−6	54
Right insula	63	4.81	33	21	12
Left middle cingulum	39	4.31	−21	−9	36
		3.92	−24	−9	27
		3.31	−18	−18	39
Left cerebellum (6)	25	3.69	−6	−72	−15
		3.56	6	−69	−9
Right cerebellum (8)	11	3.63	24	−60	−51

Note: Clusters with $p < 0.001$ (uncorrected) and a minimum size of 10 were considered statistically significant. Coordinates (XYZ) are in MNI Space.

Second, arithmetical computation and problem solving have some shared cognitive factors (e.g., central executive, general intelligence) (e.g., Andersson, 2007; Passolunghi and Pazzaglia, 2004; Swanson and Sachse-Lee, 2001; Zheng et al., 2011) but also differential cognitive correlates (e.g., Fuchs et al., 2008a, 2010; Hickendorff, 2013; Wei et al., 2012a, 2012b; Zhang et al., 2016), especially in phonological processing and semantic processing. During arithmetical computation, the phonological system supports the coding of visual or auditory numbers, as well as the temporary storage of intermediate results (e.g., Adams and Hitch, 1997; Dehaene et al., 1999; Fürst and Hitch, 2000; Logie et al., 1994). For example, Noël et al. (2001) found that the phonological similarity (but not the visual similarity) of two numbers had a major effect on both speed and accuracy for complex mental addition. In contrast, mathematical problem solving involves more semantic processing because it uses mathematical conceptual knowledge to search numerical relations among objects, events, or even abstract numbers. Mathematical word problem solving has been found to be more strongly associated with language comprehension than is arithmetical computation (Fuchs et al., 2008a; Hickendorff, 2013). Wei et al. (2012b) also found that mathematical reasoning measured with number series completion (e.g., what is the number following the number series “12:13:15:18”?) had a stronger positive correlation with sentence semantic processing ($r = 0.35$) than did arithmetical computation ($r = -0.08$).

Numerical processing and arithmetical computation in the brain

The regions around bilateral intraparietal sulcus (IPS) have been found to play a comparatively category-specific role for number processing and arithmetical computation (Arsalidou and Taylor, 2011; Dehaene et al., 2003; Eger et al., 2003; Thioux et al., 2005; Zhang et al., 2012). First, relative to language processing, number processing and arithmetical computation elicit greater activation in the parietal regions around IPS (Chochon et al., 1999; Dehaene et al., 1998, 2004, 2003; Eger

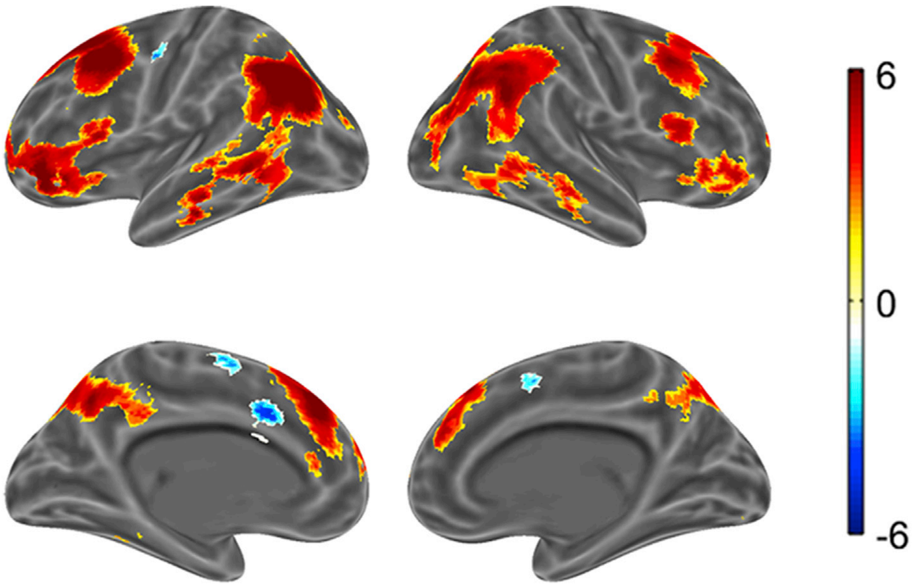

Fig. 2. Brain activations for the contrast between mathematical problem solving and arithmetical computation: Subject-wise analysis. Warm tone indicates the contrast of "mathematical problem solving – arithmetical computation". Cold tone indicates the contrast of "Arithmetical computation - mathematical problem solving". Clusters with $p < 0.001$ (uncorrected) and a spatial extent $k > 10$ voxels were considered statistically significant.

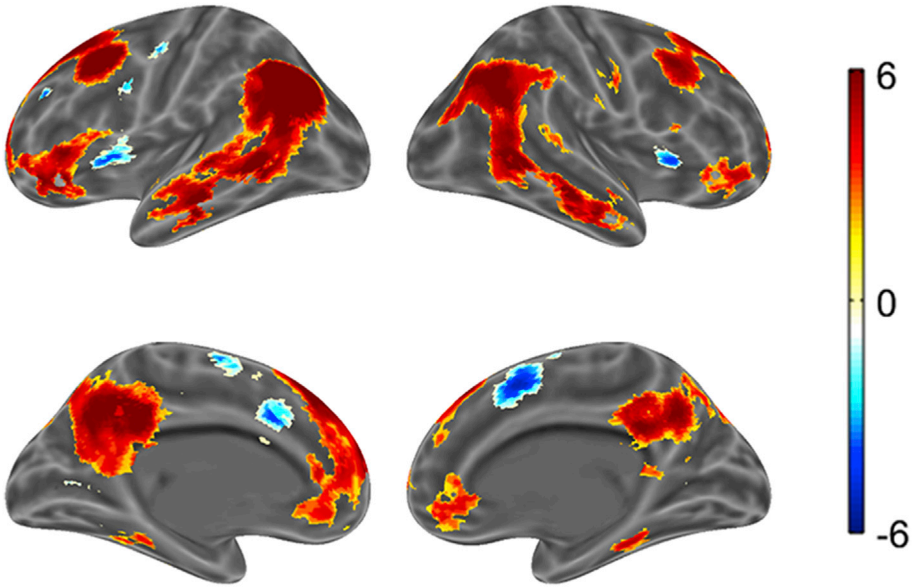

Fig. 3. Brain activations for the contrast between mathematical problem solving and arithmetical computation: Item-wise analysis. Warm tone indicates the contrast of "mathematical problem solving – arithmetical computation". Cold tone indicates the contrast of "Arithmetical computation - mathematical problem solving". Clusters with $p < 0.001$ (uncorrected) and a spatial extent $k > 10$ voxels were considered statistically significant.

et al., 2003; Kinzler and Spelke, 2007; Nieder, 2004; Thioux et al., 2005; Zhang et al., 2012). For example, Thioux et al. (2005) showed that number comparison (e.g., "Is seven larger than 5?") activated bilateral intraparietal sulcus to a greater extent than the judgment on the ferocity of animals ("Is bear more ferocious than a dog?"). Second, patients who had injuries in the left parietal cortex typically suffer from acalculia (Ashkenazi et al., 2008; Baldo and Dronkers, 2007; Delazer and Benke, 1997; Denes and Signorini, 2001; van Harskamp et al., 2002). Third, brain stimulation studies with repetitive transcranial magnetic stimulation (rTMS), transcranial direct current stimulation (tDCS), or direct cortical electrostimulation also found that stimulation administered to left and/or right parietal cortex affected number processing and arithmetical computation (Andres et al., 2005; Kadosh et al., 2010; Knops et al., 2006; Salillas et al., 2012; Sandrini et al., 2004; Semenza et al., 2016; Yu et al., 2011).

In addition to the bilateral intraparietal sulcus (for its visuo-spatial and quantity processing), the brain areas for phonological processing (e.g., the supplementary motor area, left precentral gyrus) might also be involved in numerical processing and arithmetical processing (e.g., Kong et al., 2005; Menon et al., 2000; Wu et al., 2009; Zhou et al., 2007). Previous research has shown that exact arithmetic involves more phonological or verbal processing than does approximate arithmetic (Dehaene et al., 1999). Neuropsychological investigation also showed that an infarction affecting the left frontal lobe (including the precentral gyrus) would result in difficulty in the retrieval of the multiplication table but spare addition and subtraction (Tohgi et al., 1995). As part of the phonological system, Brodmann area 6 (BA 6, including the supplementary motor area, left precentral gyrus) has been reported to be

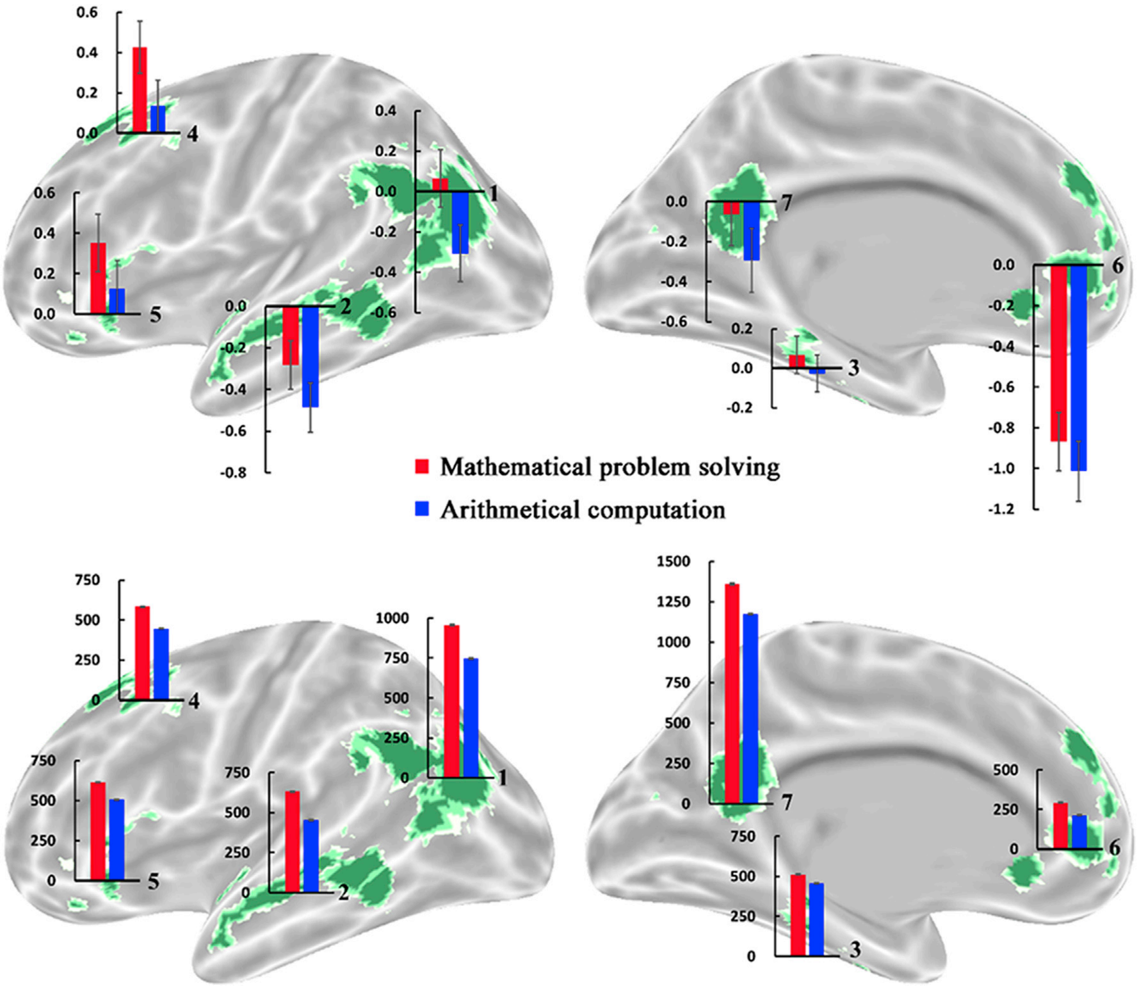

Fig. 4. Brain activations for mathematical problem solving and arithmetical computation in 7 semantic brain regions: Subject-wise analysis. The height of bars in the two brain maps of top panel means activation intensity (signal change % of BOLD), and the height of bars in the two brain maps of bottom panel means the number of positively activated voxels in $3 \times 3 \times 3$ mm^3 ($t > 0$). The 7 semantic brain regions based on a meta-analysis of 120 functional neuroimaging studies on semantic processing include: ① posterior inferior parietal lobe (angular gyrus), ② middle temporal gyrus, ③ fusiform and parahippocampal gyri, ④ dorsomedial prefrontal cortex, ⑤ inferior frontal gyrus, ⑥ ventromedial prefrontal cortex, and ⑦ posterior cingulate gyrus.

involved in complex computation (Kong et al., 2005; Menon et al., 2000; Wu et al., 2009). For example, Wu et al. (2009) showed that both continuous nine one-digit addition (e.g., $8 + 3+7 + 2+6 + 5+9 + 4+2 = ?$) and continuous nine two-digit addition activated BA 6 more than did covert reading.

In contrast to the visuo-spatial and phonological processing involved in computation, semantic processing might not be an important component of arithmetical computation. A series of studies on patients with semantic dementia showed that they could lose semantic processing ability but still preserve arithmetical performance (e.g., Butterworth and Cappelletti, 2001; Cappelletti et al., 2001; Cheng et al., 2013).

Mathematical problem solving in the brain

Although there has been much research on the neural correlates for numerical processing and arithmetical computation, few studies have been conducted for the brain organization of mathematical problem solving. The existing imaging studies of mathematical problem solving have typically focused on either the effect of problem difficulty and problem format on brain organization or the similarities and differences between mathematical problem solving and language processing. They found that problem difficulty was positively associated with activations in the dorsolateral prefrontal cortex (DLPFC) and parietal cortex (Feng et al., 2014; Jia et al., 2011; Liang et al., 2007; Prabhakaran et al., 2001; Yang et al., 2009; Zhong, 2008), perhaps mediated by general cognitive processing such as working memory (Passolunghi and Siegel, 2001; Swanson and Sachse-Lee, 2001). The studies that focused on presentation format have contrasted symbolic equation format vs. verbal story format (Sohn et al., 2004), and symbolic form vs. pictorial form (Lee et al., 2007). These studies showed that symbolic format typically activated bilateral parietal regions around IPS, but linguistic or verbal format typically elicited activation in classic language areas including left inferior frontal gyrus, posterior superior temporal gyrus, and posterior and anterior middle temporal gyrus (Monti et al., 2012; Sohn et al., 2004).

Recently two studies compared mathematical problem solving with language processing (Amalric and Dehaene, 2016; Monti et al., 2012). These studies showed a dissociation of brain organization between mathematical problem solving and language processing, which is similar to the dissociation between basic numerical processing and language processing (e.g., Butterworth and Cappelletti, 2001; Cappelletti et al., 2001; Cheng et al., 2013; Eger et al., 2003; Libertus et al., 2009; Piazza et al., 2007; Thioux et al., 2005). Specifically, mathematical processing typically activates the bilateral IPS areas and dorsal frontal areas, but language processing typically activates the middle temporal cortex (e.g., Amalric and Dehaene, 2016; Butterworth and Cappelletti, 2001; Monti et al., 2012; Thioux et al., 2005).

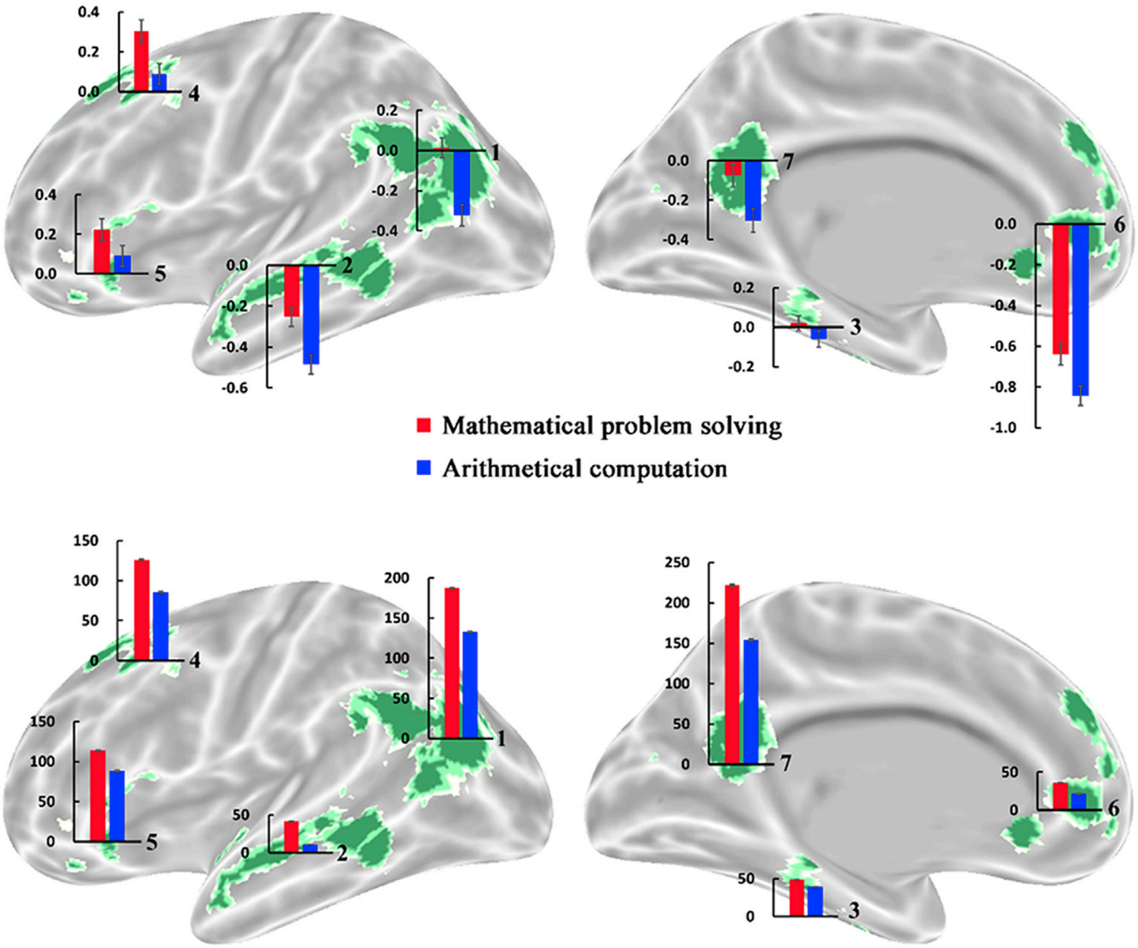

Fig. 5. Brain activations for mathematical problem solving and arithmetical computation in 7 semantic brain regions: Item-wise analysis. The height of bars in the two brain maps of top panel means activation intensity (signal change % of BOLD), and the height of bars in the two brain maps of bottom panel means the number of positively activated voxels in 3 × 3 × 3 mm^3 ($t > 0$). The 7 semantic brain regions based on a meta-analysis of 120 functional neuroimaging studies on semantic processing include: ① posterior inferior parietal lobe (angular gyrus), ② middle temporal gyrus, ③ fusiform and parahippocampal gyri, ④ dorsomedial prefrontal cortex, ⑤ inferior frontal gyrus, ⑥ ventromedial prefrontal cortex, and ⑦ posterior cingulate gyrus.

The current study

The goal of the current study was to use fMRI to identify specific neural basis for mathematical problem solving as compared to that for arithmetical computation. Mathematical problem solving was measured using number series completion, geometric problems solving, and word problems solving, which have been typically used in previous studies on mathematical problem solving (Fuchs et al., 2008b; Kintsch and Greeno, 1985; Riley and Greeno, 1988; Wei et al., 2012b). The arithmetical computation tasks used problems that were matched in their surface form to the problems for the mathematical problem solving tasks.

We hypothesized that the semantic system would be more involved in mathematical problem solving, whereas the phonological system would be more involved in arithmetical computation. The rationale for this hypothesis is as follows. First, mathematical problem solving relies on the application of conceptual/semantic knowledge (i.e., semantic processing of mathematical concepts, rules, and principles) (Fuchs et al., 2008b; Kintsch and Greeno, 1985; Riley and Greeno, 1988; Wei et al., 2012b). For example, Fuchs et al. (2008b) found that conceptual processing measured with the WJ- III Concept Formation task was correlated with mathematical problem solving (measured with word problem solving), but not with computation. Vukovic and Lesaux (2013) further showed that semantic processing (language comprehension) predicted gains in performance for data analysis/probability and geometry, but not for arithmetic or algebraic computation, after controlling for reading ability and visual-spatial working memory. Second, although there has been little brain imaging research on the role of semantic processing in mathematical problem solving, existing studies have shown that compared to simple numerical processing and arithmetical computation, mathematical semantic knowledge (e.g., mathematical terminologies and principles) elicited greater activation in the classical semantic areas (e.g., middle temporal cortex, inferior frontal gyrus) (Liu et al., 2017; Zhang et al., 2012). Such results led us to speculate that the brain areas for semantic processing would be strongly activated when mathematical semantic knowledge is used for mathematical problem solving.

The semantic processing system actually covers a wide range of brain regions (Binder et al., 2009). Binder et al. (2009) used the activation likelihood estimate technique to analyze 120 functional neuroimaging studies of semantic processing involving spoken or written word stimuli. They found a left-lateralized network comprised of 7 regions: posterior inferior parietal lobe (angular gyrus), middle temporal gyrus, fusiform and parahippocampal gyri, dorsomedial prefrontal cortex, inferior frontal gyrus, ventromedial prefrontal cortex, and posterior cingulate gyrus. We expected that this semantic processing system would be involved in mathematical problem solving.

To match the task of mathematical problem solving, we used the task of arithmetical computation with procedures (i.e., complex computation involving the application of arithmetic facts and arithmetic procedures) other than simple retrieval of arithmetic facts. We expected that arithmetical computation would rely on the phonological processing system.

Table 3

t and *p* values from the contrasts between mathematical problem solving and arithmetical computation in 7 semantic ROIs: Subject-wise analysis.

ROI	Number series	Arithmetical computation	*t*	*p*
Left angular gyrus	−0.07(0.16)	−0.41(0.17)	−4.11	0.000424
	292(2.3)	235(2.2)	−4.19	0.000354
Middle temporal gyrus	−0.41(0.13)	−0.61(0.14)	−3.34	0.002811
	167(2.5)	122(2.3)	−2.08	0.049161
Fusiform and parahippocampal gyri	0.00(0.11)	−0.12(0.11)	−3.27	0.003331
	148(1.7)	130(1.8)	−1.95	0.063257
Dorsomedial prefrontal cortex	0.29(0.14)	0.06(0.16)	−3.22	0.003831
	178(1.7)	141(1.8)	−3.88	0.000755
Inferior frontal gyrus (orbital and triangle)	0.25(0.15)	0.02(0.16)	−2.89	0.008176
	193(1.8)	157(1.8)	−2.82	0.009662
Ventromedial prefrontal cortex	−0.86(0.15)	−0.98(0.16)	−2.12	0.045222
	99(2.0)	80(2.1)	−1.86	0.076431
Post cingulum gyrus	−0.17(0.18)	−0.41(0.18)	−2.72	0.012228
	418(3.3)	355(3.2)	−1.89	0.072076
ROI	Geometric problems	Arithmetical computation	*t*	*p*
Left angular gyrus	0.19(0.15)	−0.22(0.14)	−4.76	8.40E-05
	341(2.2)	271(2.0)	−4.12	0.000413
Middle temporal gyrus	−0.26(0.13)	−0.43(0.13)	−3.14	0.004645
	224(2.6)	166(2.5)	−3.02	0.006086
Fusiform and parahippocampal gyri	0.16(0.11)	0.08(0.10)	−1.74	0.095495
	196(1.8)	183(1.7)	−1.02	0.319141
Dorsomedial prefrontal cortex	0.47(0.14)	0.18(0.12)	−3.73	0.00109
	201(1.6)	154(1.4)	−4.42	0.0002
Inferior frontal gyrus (orbital and triangle)	0.34(0.14)	0.18(0.14)	−2.2	0.038441
	204(1.7)	172(1.7)	−3.23	0.003739
Ventromedial prefrontal cortex	−0.89(0.14)	−1.00(0.14)	−1.83	0.080814
	92(1.7)	68(1.9)	−2.65	0.014386
Post cingulum gyrus	0.16(0.17)	−0.07(0.15)	−1.84	0.07847
	522(3.0)	466(2.9)	−1.67	0.10848
ROI	Arithmetic word problems	Arithmetical computation	*t*	*p*
Left angular gyrus	0.07(0.15)	−0.30(0.14)	−4.77	8.20E-05
	323(2.2)	242(2.1)	−4.75	8.70E-05
Middle temporal gyrus	−0.18(0.13)	−0.42(0.12)	−3.01	0.006224
	240(2.6)	166(2.4)	−2.77	0.010805
Fusiform and parahippocampal gyri	0.05(0.11)	−0.04(0.10)	−1.94	0.064832
	168(1.8)	144(1.8)	−2.16	0.041438
Dorsomedial prefrontal cortex	0.52(0.14)	0.16(0.13)	−4.67	0.000106
	205(1.6)	151(1.6)	−5.46	1.50E-05
Inferior frontal gyrus (orbital and triangle)	0.47(0.16)	0.18(0.15)	−3.67	0.001275
	217(1.7)	177(1.8)	−3.13	0.004656
Ventromedial prefrontal cortex	−0.86(0.15)	−1.06(0.15)	−3.12	0.004785
	102(2.0)	66(1.9)	−4.1	0.000443
Post cingulum gyrus	−0.19(0.16)	−0.40(0.17)	−2.13	0.044481
	419(3.0)	352(3.2)	−2.23	0.035459

Note. For each ROI, the values in the first row refer to signal change (%), the values in the second row refer to the number of positively activated voxels ($p > 0$).

Table 4

t and *p* values from the contrasts between mathematical reasoning and arithmetical computation in 7 semantic ROIs: Item-wise analysis.

ROI	Number series	Arithmetical computation	*t*	*p*
Left angular gyrus	0.12(0.08)	−0.24(0.19)	−5.82	2.70E-07
	217(1.3)	155(1.5)	−4.12	0.00012
Middle temporal gyrus	−0.18(0.07)	−0.42(0.08)	−5.04	4.90E-06
	45(1.2)	19(0.9)	−2.76	0.007677
Fusiform and parahippocampal gyri	0.11(0.06)	0.03(0.07)	−2.4	0.019599
	75(1.1)	59(1.2)	−1.49	0.140536
Dorsomedial prefrontal cortex	0.34(0.10)	0.10(0.10)	−3.14	0.002662
	133(1.3)	97(1.2)	−2.81	0.006742
Inferior frontal gyrus (orbital and triangle)	0.21(0.08)	0.12(0.09)	−1.68	0.098821
	106(1.2)	94(1.2)	−1.07	0.28738
Ventromedial prefrontal cortex	−0.62(0.09)	−0.89(0.09)	−3.85	0.000296
	37(0.8)	25(0.7)	−2.68	0.00961
Post cingulum gyrus	0.14(0.09)	−0.16(0.11)	−3.67	0.000523
	302(1.9)	225(2.1)	−2.43	0.018273
ROI	Geometric problems	Arithmetical computation	*t*	*p*
Left angular gyrus	−0.02(0.08)	−0.36(0.09)	−5.54	7.80E-07
	179(1.3)	127(1.2)	−4.3	6.70E-05
Middle temporal gyrus	−0.21(0.07)	−0.44(0.08)	−5.23	2.40E-06
	58(1.2)	11(0.7)	−5.15	3.20E-06
Fusiform and parahippocampal gyri	−0.05(0.06)	−0.11(0.07)	−2.07	0.043375
	35(0.7)	32(0.7)	−0.92	0.363895
Dorsomedial prefrontal cortex	0.39(0.08)	0.02(0.09)	−6.49	2.10E-08
	144(1.2)	82(1.0)	−6.29	4.50E-08
Inferior frontal gyrus (orbital and triangle)	0.36(0.09)	0.07(0.19)	−4.45	3.90E-05
	139(1.3)	91(1.2)	−4.11	0.000124
Ventromedial prefrontal cortex	−0.76(0.08)	−0.90(0.07)	−2.89	0.005458
	37(0.7)	19(0.7)	−4.51	3.20E-05
Post cingulum gyrus	−0.28(0.10)	−0.45(0.09)	−2.53	0.014095
	162(1.8)	110(1.4)	−2.48	0.016219
ROI	Arithmetic word problems	Arithmetical computation	*t*	*p*
Left angular gyrus	−0.04(0.09)	−0.36(0.08)	−5.41	9.60E-07
	169(1.3)	117(1.2)	−4.05	0.00014
Middle temporal gyrus	−0.35(0.08)	−0.58(0.08)	−4.56	2.30E-05
	24(1.1)	3(0.5)	−2.94	0.004484
Fusiform and parahippocampal gyri	0.00(0.06)	−0.10(0.06)	−3.23	0.001962
	37(0.7)	27(0.7)	−2.45	0.016785
Dorsomedial prefrontal cortex	0.19(0.10)	0.13(0.08)	−0.89	0.37867
	104(1.1)	78(1.0)	−2.73	0.008198
Inferior frontal gyrus (orbital and triangle)	0.12(0.10)	0.09(0.09)	−0.37	0.712251
	98(1.2)	81(1.0)	−1.71	0.091702
Ventromedial prefrontal cortex	−0.55(0.09)	−0.75(0.08)	−3.61	0.0006
	35(0.9)	21(0.6)	−2.93	0.004608
Post cingulum gyrus	−0.09(0.09)	−0.30(0.09)	−3.14	0.002532
	204(1.8)	130(1.8)	−2.69	0.009111

Note. For each ROI, the first row is related to signal change (%), and the second row is related to the number of positively activated voxels ($p > 0$).

Arithmetical computation has been shown to rely on phonological processing (Adams and Hitch, 1997; Dehaene et al., 1999; Fürst and Hitch, 2000; Logie et al., 1994). For example, Fürst and Hitch (2000) found that mental addition was disrupted by phonological suppression. Multi-step arithmetical computation would involve phonological representation for the intermediate results (Campbell and Charness, 1990). Vukovic and Lesaux (2013) found that phonological skills measured with phonological decoding were involved in conventional arithmetic.

In this study, we examined the imaging data in two ways. First, we directly contrasted the brain activation for mathematical problem solving and arithmetical computation. Second, we conducted an ROI analysis. ROIs for mathematical problem solving were defined based on the meta-analysis of 120 functional neuroimaging studies on word-based semantic processing (Binder et al., 2009). ROIs for arithmetical computation were defined according to the contrasts between phonological processing and semantic processing (Gold et al., 2005; Gold and Buckner, 2002; Poldrack et al., 1999; Price et al., 1997) and the contrast between multiplication and addition (Zhou et al., 2007). Only one region, Brodmann area 6 (BA 6), was found to be common across the four studies for phonological processing.

Methods

Participants

Twenty-four healthy right-handed university students (11 males and 13 females) were recruited from Beijing Normal University, China. They majored in a wide range of disciplines except for mathematics. The average age of the participants was 21.5 years, ranging from 18.7 to 26.6 years. They self-reported having normal or corrected-to-normal eyesight

Table 5
t and p values from the contrasts between mathematical problem solving and arithmetical computation in 2 ROIs: Subject-wise analysis.

ROI	Number series	Arithmetical computation	t	p
Supplementary motor area	0.22(0.17)	0.43(0.18)	2.32	0.029757
	55(1.1)	66(1.1)	2.56	0.017429
Precentral gyrus	0.18(0.12)	0.32(0.13)	2.57	0.017304
	76(1.1)	87(1.1)	2.09	0.047489
ROI	Geometric problems	Arithmetical computation	t	p
Supplementary motor area	0.25(0.18)	0.58(0.19)	2.99	0.006468
	56(1.1)	64(1.2)	1.64	0.113605
Precentral gyrus	0.22(0.11)	0.34(0.11)	2.76	0.011139
	81(1.0)	89(1.0)	1.85	0.077312
ROI	Arithmetic word problems	Arithmetical computation	t	p
Supplementary motor area	0.51(0.19)	0.48(0.18)	−0.37	0.716131
	63(1.1)	65(1.1)	0.62	0.543141
Precentral gyrus	0.23(0.13)	0.23(0.12)	0.02	0.984457
	76(1.1)	80(1.1)	1.00	0.326693

Note. For each ROI, the values in the first row refer to signal change (%), the values in the second row refer to the number of positively activated voxels ($p > 0$).

Table 6
t and p values from the contrasts between mathematical problem solving and arithmetical computation in 2 ROIs: Item-wise analysis.

ROI	Number series	Arithmetical computation	t	p
Supplementary motor area	0.21(0.07)	0.48(0.09)	4.62	0.000064
	20(0.7)	43(0.7)	4.88	0.000031
Precentral gyrus	0.21(0.07)	0.44(0.08)	4.69	0.000052
	39(0.8)	76(0.9)	5.71	0.000003
ROI	Geometric problems	Arithmetical computation	t	p
Supplementary motor area	0.20(0.07)	0.36(0.07)	3.54	0.001461
	22(0.7)	37(0.7)	3.40	0.002116
Precentral gyrus	0.20(0.07)	0.28(0.07)	1.94	0.063046
	54(0.9)	65(1.0)	1.46	0.156783
ROI	Arithmetic word problems	Arithmetical computation	t	p
Supplementary motor area	0.28(0.07)	0.22(0.06)	−2.00	0.056038
	28(0.7)	26(0.7)	−0.55	0.583546
Precentral gyrus	0.17(0.05)	0.19(0.07)	0.65	0.524117
	42(0.8)	49(0.9)	1.39	0.176930

Note. For each ROI, the values in the first row refer to signal change (%), the values in the second row refer to the number of positively activated voxels ($p > 0$).

and normal hearing. Participants had no brain abnormality on their T1-weighted high-resolution magnetic resonance images (MRI) as determined by a neuroradiologist. Informed written consent was obtained from each participant after procedures were fully explained. The experiment was approved by State Key Laboratory of Cognitive Neuroscience and Learning at Beijing Normal University. Participants were compensated 100 RMB for their time.

Material

Three types of mathematical problem solving were included in this study: number series completion, mathematical word problem solving, and geometric problem solving. Each type of mathematical problem solving was matched with arithmetical computation that used a similar presentation format.

Number series completion vs. arithmetical computation

Number series completion involved the identification of a hidden rule underlying a number series. For example, given the number series “12:13:15:18”, participants needed to identify the rule “a, a+1, a+2, a+3” in order to find the subsequent number, a+4. Twelve types of rules were used (Zhang et al., 2016). To match the visual stimuli, arithmetical computation used the same number series, but the task was to sum the numbers (e.g., “13:18:12:15” and the answer is 58).

Geometric problem solving vs. arithmetical computation

Geometric problems involved numbers and geometric symbols, without any words. The numbers in the geometric problems indicated the amount or the relationships of segments, angles, perimeter, or area. A question mark indicated the value needed to be determined. For each geometric problem, participants needed to recall at least two geometric rules, but related computation was simple. For example, the geometric problem as shown in Fig. 1 involves rules “Each interior angle for an equilateral triangle is 60°” and “The central angle is twice the peripheral angle”. To match the visual stimuli involved, arithmetical computation problems also used geometric problems, but they involved little problem solving. For example, the arithmetical computation problem in Fig. 1 just asked participants to calculate the angle. This problem only involved the simple rule “The sum of all interior angles in a triangle is 180°”, with the computation as “180-84-69”.

Mathematical word problems vs. arithmetical computation

For mathematical word problems, participants needed to perform at least two steps of numerical reasoning to determine the answer, although little effort was needed for arithmetical computation. For example, the problem was “Lucy has 90 marbles. Her brother has 60 marbles. How many marbles must she give to him so both of them will have the same amount?”, and the correct answer was 15, calculated from “(90-60) ÷2”. To match the text of word problems, arithmetical computation problems were presented as mathematical word problems with less reasoning but more complex computation. For example, an arithmetical computation problem was “Lucy has 146 marbles. Her brother has 68 marbles. How many more marbles does she have than her brother?” The computation is “146-68”.

In sum, the tasks of arithmetical computation and mathematical problem solving shared similar surface forms (visual angle, figures, and mathematical symbols) and similar amounts of language involvement, but they had different amounts of arithmetical computation and mathematical reasoning. Functional neuroimaging measurements are very sensitive to differences in response time and accuracy between tasks (Binder et al., 2004; Braver et al., 1997; Jonides et al., 2006; Tregellas et al., 2006). Therefore, we conducted several rounds of pilot experiments to select materials that matched the difficulty levels between mathematical problem solving tasks and their corresponding arithmetical computation tasks. The procedure used in the pilot experiments were similar to that used in the current study except it was out of the scanner. About 30 undergraduates took part in the pilot experiments, and none of them participated in the formal experiment. Based on the pilot experiments, we eventually selected 188 problems: 34 number series completion problems and 34 matched arithmetical computation problems, 30 geometric problems and 30 matched arithmetical computation problems, and 30 mathematical word problems and 30 matched arithmetical computation problems.

Each problem was presented in the upper middle part of the screen with four candidate answers in the lower part of the screen. The four candidate answers were divided into two on the left side of the screen and two on the right side. Participants needed to determine whether the correct answer was on the left or the right side. The four candidate answers were used to reduce the possibilities of guessing, which is typically based on some characteristics of the candidate answers.

Apparatus and imaging parameters

Functional MRI was carried out on a Siemens (Munich, Germany) 3T Trio scanner using an 8-channel phase array head coil. Participants laid supine in the scanner with their heads immobilized. A single shot, T2*-weighted gradient-echo echo planar imaging (EPI) sequence was used for the fMRI scans, with slice thickness of 6 mm and no gap between slices, in-plane resolution of 3.75 × 3.75 mm, and TR/TE = 3000 ms/30 ms. The field of view was 240 × 240 mm, and the acquisition matrix was 64 × 64. Thirty contiguous axial slices parallel to AC-PC were acquired. Three hundred and sixty two images were acquired with a total scan time of 724 s in a single run. Additionally, high-resolution T1-weighted anatomical images were acquired for each participant (three-dimensional, gradient-echo pulse-sequence, TR/TE = 25 ms/6 ms, FOV = 220 × 220 mm, 89–92 contiguous slices, matrix = 220 × 220, and thickness = 2 mm).

fMRI scanning procedures

Before the fMRI scanning, participants were given a practice session outside of the scanner. In the scanner, mathematical problems were projected onto the center of a translucent screen and viewed by the participants through a mirror attached to the head coil. The stimuli were presented in black against a white background. The visual angle of each problem was less than 3° in both horizontal and vertical directions.

There were four scanning runs: a run for structural scanning, and other three runs for functional scanning. One run was for number series completion and corresponding arithmetical computation, a second run for geometric problem solving and corresponding arithmetical computation, and a third run for mathematical word problem solving and corresponding arithmetical computation. The order of the three runs was counterbalanced across participants. There were a total of 16 blocks per run: 4 blocks for mathematical problem solving (each lasting 1 min), 4 blocks for corresponding arithmetical computation (each lasting 1 min), and 8 blocks of visual fixation (each lasting 30 s). Before each block, there was a cue lasting 2 s to remind participants of the task to come. Within each run, mathematical problems were presented in a random order.

Participants were encouraged to respond as quickly and accurately as possible by using their left or right index finger to press a button. After the participant responded, a new problem was presented after a blank of 1000 ms. If the participants failed to respond within 20 s, the problem would disappear and a new one was presented. If there were 8 s or less left in a block, instead of presenting a new question, a fixation sign ("+") was presented during the rest of the block.

Image preprocessing

Neuroimage preprocessing was performed using SPM12 (Statistical Parametric Mapping, Welcome Department of Cognitive Neurology, London, http://www.fil.ion.ucl.ac.uk/spm/). The functional data set acquired from the experiment consisted of 362 image volumes for each of the three functional runs. Functional images were realigned to the first volume in the scanning session using affine transformations. A mean functional image volume was constructed for each participant from the realigned image volumes. This mean image volume was then used to determine parameters for spatial normalization. The normalization parameters were then applied to the corresponding functional image volumes for each participant. Spatial smoothing was performed on the normalized functional images using a Gaussian kernel 8 mm FWHM (full width half maximum).

For the subject-wise analysis, statistical analyses were conducted on the smoothed data using a boxcar design with a canonical HRF (hemodynamic response function). A high-pass filter (186 s) was applied in order to remove low frequency effects and a low-pass filter (4 s) to remove the high frequency noise. The global temporal trend was removed. Contrasts of interests were calculated for each individual (see below) and subsequently entered into random effects analyses at the subject-based group level using one-sample t tests.

For the item-wise analysis, statistical analyses were conducted on the smoothed data using a single trial design with a canonical HRF (hemodynamic response function). A high-pass filter (186 s) was applied in order to remove low frequency effects and a low-pass filter (4 s) to remove the high frequency noise. The global temporal trend was removed. Contrasts of interests were calculated for each item and subsequently entered into random effects analyses at the item-based group level using one-sample t tests.

For all contrasts of brain activation, a lenient threshold of $p < 0.001$ with a minimum cluster size of 10 voxels was used. Further ROI analysis used ROIs defined based on the meta-analysis of 120 functional neuroimaging studies on semantic processing (Binder et al., 1999) and two ROIs (i.e., the supplementary motor area and precentral gyrus) defined based on the contrast of phonological processing and semantic processing (Gold et al., 2005; Gold and Buckner, 2002; Poldrack et al., 1999; Price et al., 1997).

Results

Behavioral results

The mean reaction times (RTs) were 5859 ms (SD = 1285) for number series completion and 5740 ms (SD = 920) for corresponding numerical computation, 8164 ms (SD = 1405) for geometric problem solving and 8242 (SD = 961) for corresponding numerical computation, and 8485 ms (SD = 1574) for arithmetic word problem solving and 8163 ms (SD = 1683) for corresponding numerical computation. Repeated measures analysis of variance (ANOVA) with type of mathematical processing (mathematical problem solving vs. arithmetical computation) and problem type (number series vs. geometric problems vs. word problems) showed no significant effect of type of mathematical processing, $F(1, 23) = 0.77$, $p = 0.39$, $\eta^2 = 0.03$, and no interaction between type of mathematical processing and problem type, $F\,(2, 46) = 1\ 0.15$, $p = 0.33$, $\eta^2 = 0.05$. The main effect of problem type was significant, $F\,(2, 46) = 89.97$, $p < 0.001$, $\eta^2 = 0.80$, with shorter RT for number series than geometric problems ($p < 0.001$) and word problems ($p < 0.001$).

The mean error rates are 14.1% (SD = 9.2) for number series completion and 15.1% (SD = 6.9) for corresponding numerical computation, 19.2% (SD = 10.2) for geometric problem solving and 16.3% (SD = 9.5) for corresponding numerical completion, and 12.6% (SD = 7.2) for arithmetic word problem solving and 12.5% (SD = 7.6) for corresponding numerical computation. Repeated measures ANOVA (same as that for RT) showed no main effect of the type of mathematical processing, $F\,(1, 23) = 0.27$, $p = 0.61$, $\eta^2 = 0.01$, and no interaction between type of mathematical processing and problem type, $F\,(2, 46) = 0.82$, $p = 0.44$, $\eta^2 = 0.04$. The main effect of problem type was significant, $F\,(2, 46) = 5.09$, $p = 0.01$, $\eta^2 = 0.18$, with lower error rates for word problems than geometric problems ($p < 0.01$).

Both RT and error rate results suggest that we were able to match satisfactorily the mathematical problem solving tasks with their corresponding arithmetical computation tasks in terms of task difficulty.

Contrasts between mathematical problem solving and arithmetical computation

The contrasts between mathematical problem solving and arithmetical computation based on subject-wise analyses across the three types of problems are shown in Table 1 and Fig. 1, and the contrasts based on item-wise analyses are shown in Table 2 and Fig. 2. Both sets of analyses showed very similar activation patterns. Mathematical problem solving elicited greater activations than did arithmetical computation in the left hemisphere, including the angular gyrus, middle temporal gyrus, fusiform and parahippocampal gyri, dorsomedial prefrontal cortex, inferior frontal gyrus (including the triangle and orbital inferior gyrus), ventromedial prefrontal cortex, and posterior cingulate gyrus.

In contrast, both the subject- and item-wise analyses showed that arithmetical computation elicited greater activation in bilateral precentral gyrus, supplementary motor area, left insula, and left superior temporal gyrus (see Tables 1 and 2, Figs. 2 and 3).

ROI analysis

Seven ROIs for semantic processing and two ROIs for phonological processing were used. The intensity and volume of activations were extracted from the ROIs. Fig. 4 (subject-wise analysis) and Fig. 5 (item-wise analysis) show the results combined across problem types and Table 3 shows the results by problem type. Overall, results showed that mathematical problem solving elicited activation with greater intensity and larger volume than did arithmetical computation for each of the 7 semantic ROIs (see Table 4).

The intensity and volume of activations were also extracted from the two phonological ROIs. Table 5 shows the results by problem type. Overall, results showed that arithmetical computation had greater intensity and/or larger volume than did number series problem solving and geometric problem solving for both phonological ROIs. No significant differences between arithmetical computation and word problem solving were found (see Table 6).

Discussion

The current study aimed to investigate the brain system for mathematical problem solving as compared with that for arithmetical computation. Three types of mathematical problem solving were used: number series completion, geometric problem solving, and arithmetic word problem solving. Similar to Binder et al.'s (2009) emphasis on good designs of tasks for imaging studies, we carefully designed the contrasts between mathematical problem solving and arithmetical computation. First, there was a good match between the two types of tasks in terms of phonological and orthographic processes: the same visual stimuli and similar extent of language processing for mathematical problem solving and arithmetical computation for each problem type. Second, task difficulty (in terms of both RT and error rates) was also matched between the two types of mathematical processing. Third, we used an active control task (arithmetical computation) rather than a passive task (e.g., being in resting state, fixating a point in the visual field, or focusing on the scanner sounds). Fourth, multiple (three in the current study) types of contrasts were used in the current investigation.

Consistent with our predictions, mathematical problem solving was subserved by a semantic network consisting of 7 brain regions. Results were consistent regardless of the method of analysis (subject- or item-wise analysis, whole-brain or ROI analyses). In contrast, arithmetical computation was typically subserved by BA6 (including the supplementary motor area and precentral gyrus).

The important role of the semantic network in mathematical problem solving makes sense because much research has shown an important role of conceptual knowledge in mathematical problem solving (Fuchs et al., 2008a; Kintsch and Greeno, 1985; Niemi, 1996; Riley and Greeno, 1988; Wei et al., 2012b). In contrast, the semantic network is less important for arithmetical computation. For example, semantic dementia patients tend to preserve numerical processing (Butterworth and Cappelletti, 2001; Cappelletti et al., 2001, 2005; Diesfeldt, 1993).

The semantic network from Binder et al.'s meta-analysis (Binder et al., 2009) was based on neuroimaging studies using spoken or written word stimuli. The arithmetic word problems in our study also involved words, but the number series and geometric problems did not involve any words, except for Arabic digits. Our results seem to suggest that the semantic network is not limited to the processing of word stimuli. Instead, it seems to process Arabic digits as well as geometric shapes. Perhaps this network processes artificial symbols in general. Moreover, as Binder et al. (2009) stated, the semantic system is strikingly similar to the "default network''(Binder et al., 1999; Mazoyer et al., 2001; McKiernan et al., 2003; Raichle et al., 2001; Shulman et al., 1997). In our study, mathematical problem solving also had deactivation in the default network. This deactivation was however smaller than that for arithmetical computation, perhaps due to the greater involvement of semantic processing in mathematical problem solving.

As expected, arithmetical computation had greater activation than did mathematical problem solving in the brain areas for phonological processing including BA 6. Previous research has shown that BA 6 is involved in complex computation (Kong et al., 2005; Menon et al., 2000; Wu et al., 2009; Zago et al., 2001). The role of BA 6 in phonological processing has also been shown by neuroimaging studies of language processing (Gold et al., 2005; Gold and Buckner, 2002; Poldrack et al., 1999; Price et al., 1997).

Our results have important theoretical implications. They furthered our understanding of the relation between language processing and mathematical processing. Whereas previous studies emphasized the dissociation between mathematical and language processing (Amalric and Dehaene, 2016; Monti et al., 2012), our results highlighted the close association between one component of mathematical processing (problem solving) and language processing. Rather than viewing these results as contradicting one another, we think they present a more nuanced picture of the neural bases of mathematical processing. Specifically, there is clear evidence of a dissociation between basic numerical processing and language processing as shown by both neuroimaging (e.g., Eger et al., 2003; Libertus et al., 2009; Piazza et al., 2007; Thioux et al., 2005) and neuropsychological studies (e.g., Butterworth and Cappelletti, 2001; Cappelletti et al., 2001; Cheng et al., 2013). The dissociation, however, does not mean a complete separation of mathematical processing from language processing. Depending on the type of mathematical processing, the involvement of the semantic system can vary. For example, simple addition or subtraction involves more semantic processing (i.e., greater activations in bilateral inferior parietal cortex) than multiplication (e.g., Prado et al., 2011; Zhou et al., 2006, 2007, 2009), which in turn involves more semantic processing than number comparison (Dehaene, 1996). This study showed that mathematical problem solving (all three types measured in this study) involved more semantic processing than did arithmetical computation.

Conclusion

The current study showed that the semantic network in the brain subserved mathematical problem solving. Relevant brain regions included posterior inferior parietal lobe (angular gyrus), middle temporal gyrus, fusiform and parahippocampal gyri, dorsomedial prefrontal cortex, inferior frontal gyrus, ventromedial prefrontal cortex, and posterior cingulate gyrus.

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References

Adams, J.W., Hitch, J.Z., 1997. Working memory and Children's mental addition. J. Exp. Child. Psychol. 67, 21–38.

Amalric, M., Dehaene, S., 2016. Origins of the brain networks for advanced mathematics in expert mathematicians. Proc. Natl. Acad. Sci. U. S. A. 113, 4909–4917.

Andersson, U., 2007. The contribution of working memory to children's mathematical word problem solving. Appl. Cogn. Psychol. 21, 1201–1216.

Andres, M., Seron, X., Olivier, E., 2005. Hemispheric lateralization of number comparison. Brain Res. Cogn. Brain Res. 25, 283–290.

Arsalidou, M., Taylor, M.J., 2011. Is 2+2=4? Meta-analyses of brain areas needed for numbers and calculations. Neuroimage 54, 2382–2393.

Ashkenazi, S., Henik, A., Ifergane, G., Shelef, I., 2008. Basic numerical processing in left intraparietal sulcus (IPS) acalculia. Cortex 44, 439–448.

Baldo, J.V., Dronkers, N.F., 2007. Neural correlates of arithmetic and language comprehension: a common substrate? Neuropsychologia 45, 229–235.

Binder, J.R., Desai, R.H., Graves, W.W., Conant, L.L., 2009. Where is the semantic system? A critical review and meta-analysis of 120 functional neuroimaging studies. Cereb. Cortex 19, 2767–2796.

Binder, J.R., Frost, J.A., Hammeke, T.A., Bellgowan, P.S.F., Rao, S.M., Cox, R.W., 1999. Conceptual processing during the conscious resting state: a functional MRI study. J. Cogn. Neurosci. 11, 80–93.

Binder, J.R., Liebenthal, E., Possing, E.T., Medler, D.A., Ward, B.D., 2004. Neural correlates of sensory and decision processes in auditory object identification. Nat. Neurosci. 7, 295–301.

Braver, T.S., Cohen, J.D., Nystrom, L.E., Jonides, J., Smith, E.E., Noll, D.C., 1997. A parametric study of prefrontal cortex involvement in human working memory. Neuroimage 5, 49–62.

Butterworth, B., Cappelletti, M., 2001. Category specificity in reading and writing: the case of number words. Nat. Neurosci. 4, 784–786.

Campbell, J.I.D., Charness, N., 1990. Age-related declines in working -memory skills-evidence from a complex calculation task. Dev. Psychol. 26, 879–888.

Cappelletti, M., Butterworth, B., Kopelman, M., 2001. Spared numerical abilities in a case of semantic dementia. Neuropsychologia 39, 1224–1239.

Cappelletti, M., Kopelman, M.D., Morton, J., Butterworth, B., 2005. Dissociations in numerical abilities revealed by progressive cognitive decline in a patient with semantic dementia. Cogn. Neuropsychol. 22, 771–793.

Cheng, D., Zhou, A., Yu, X., Chen, C., Jia, J., Zhou, X., 2013. Quantifier processing can be dissociated from numerical processing: evidence from semantic dementia patients. Neuropsychologia 51, 2172–2183.

Chochon, F., Cohen, L., Van De Moortele, P., Dehaene, S., 1999. Differential contributions of the left and right inferior parietal lobules to number processing. J. Cogn. Neurosci. 11, 617–630.

Dehaene, S., 1996. The organization of brain activations in number comparison: event-related potentials and the additive-factors method. J. Cogn. Neurosci. 8, 47–68.

Dehaene, S., Dehaene-Lambertz, G., Cohen, L., 1998. Abstract representations of numbers in the animal and human brain. Trends Neurosci. 21, 355–361.

Dehaene, S., Molko, N., Cohen, L., Wilson, A.J., 2004. Arithmetic and the brain. Curr. Opin. Neurobiol. 14, 218–224.

Dehaene, S., Piazza, M., Pinel, P., Cohen, L., 2003. Three parietal circuits for number processing. Cogn. Neuropsychol. 20, 487–506.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., Tsivkin, S., 1999. Sources of mathematical thinking: behavioral and brain-imaging evidence. Science 284, 970–974.

Delazer, M., Benke, T., 1997. Arithmetic facts without meaning. Cortex 33, 697–710.

Denes, G., Signorini, M., 2001. Door but not four and 4 a category specific transcoding deficit in a pure acalculic patient. Cortex 37, 267–277.

Diesfeldt, H.F.A., 1993. Progressive decline of semantic memory with preservation of number processing and calculation. Behav. Neurol. 6, 239–242.

Eger, E., Sterzer, P., Russ, M.O., Giraud, A.-L., Kleinschmidt, A., 2003. A supramodal number representation in human intraparietal cortex. Neuron 37, 719–726.

Epelboim, J., Suppes, P., 2001. A model of eye movements and visual working memory during problem solving in geometry. Vis. Res. 41, 1561–1574.

Feng, X., Peng, L., Chang-Quan, L., Yi, L., Hong, L., 2014. Relational complexity modulates activity in the prefrontal cortex during numerical inductive reasoning: an fMRI study. Biol. Psychol. 101, 61–68.

Fuchs, L.S., Fuchs, D., Hamlett, C.L., Lambert, W., Stuebing, K., Fletcher, J.M., 2008a. Problem solving and computational skill: are they shared or distinct aspects of mathematical cognition? J. Educ. Psychol. 100, 30–47.

Fuchs, L.S., Geary, D.C., Compton, D.L., Fuchs, D., Hamlett, C.L., Seethaler, P.M., Bryant, J.D., Schatschneider, C., 2010. Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? Dev. Psychol. 46, 1731–1746.

Fuchs, L.S., Geary, D.C., Fuchs, D., Compton, D.L., Hamlett, C.L., 2014. Sources of individual differences in emerging competence with numeration understanding versus multidigit calculation skill. J. Educ. Psychol. 106, 482–498.

Fuchs, L.S., Seethaler, P.M., Powell, S.R., Fuchs, D., Hamlett, C.L., Fletcher, J.M., 2008b. Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. Except. Child. 74, 155–173.

Fürst, AJ., Hitch, G.J., 2000. Separate roles for executive and phonological components of working memory in mental arithmetic. Mem. Cognit. 28, 774–782.

Geary, D.C., Saults, S.J., Liu, F., Hoard, M.K., 2000. Sex differences in spatial cognition, computational fluency, and arithmetical reasoning. J. Exp. Child. Psychol. 77, 337–353.

Giofrè, D., Mammarella, I.C., Ronconi, L., Cornoldi, C., 2013. Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. Learn. Individ. Differ. 23, 114–122.

Gold, B.T., Balota, D.A., Kirchhoff, B.A., Buckner, R.L., 2005. Common and dissociable activation patterns associated with controlled semantic and phonological processing: evidence from FMRI adaptation. Cereb. Cortex 15, 1438–1450.

Gold, B.T., Buckner, R.L., 2002. Common prefrontal regions coactivate with dissociable posterior regions during controlled semantic and phonological tasks. Neuron 35, 803–812.

Hickendorff, M., 2013. The language factor in elementary mathematics assessments: computational skills and applied problem solving in a multidimensional irt framework. Appl. Meas. Educ. 26, 253–278.

Inglis, M., Attridge, N., Batchelor, S., Gilmore, C., 2011. Non-verbal number acuity correlates with symbolic mathematics achievement: but only in children. Psychon. Bull. Rev. 18, 1222–1229.

Jia, X., Liang, P., Lu, J., Yang, Y., Zhong, N., Li, K., 2011. Common and dissociable neural correlates associated with component processes of inductive reasoning. Neuroimage 56, 2292–2299.

Jonides, J., Nee, D.E., Berman, M.G., 2006. What has functional neuroimaging told us about the Mind? So many examples, so little space. Cortex 42, 414–417.

Kadosh, R.C., Muggleton, N., Silvanto, J., Walsh, V., 2010. Double dissociation of format-dependent and number-specific neurons in human parietal cortex. Cereb. Cortex 20, 2166–2171.

Kintsch, W., Greeno, J.G., 1985. Understanding and solving word arithmetic problems. Psychol. Rev. 92, 109–129.

Kinzler, K.D., Spelke, E.S., 2007. Core systems in human cognition. Prog. Brain Res. 164, 257–264.

Knops, A., Nuerk, H.C., Sparing, R., Foltys, H., Willmes, K., 2006. On the functional role of human parietal cortex in number processing: how gender mediates the impact of a 'virtual lesion' induced by rTMS. Neuropsychologia 44, 2270–2283.

Kong, J., Wang, C., Kwong, K., Vangel, M., Chua, E., Gollub, R., 2005. The neural substrate of arithmetic operations and procedure complexity. Brain Res. Cogn. Brain Res. 22, 397–405.

Lee, K., Lim, Z.Y., Yeong, S.H., Ng, S.F., Venkatraman, V., Chee, M.W., 2007. Strategic differences in algebraic problem solving: neuroanatomical correlates. Brain Res. 1155, 163–171.

Liang, P., Zhong, N., Lu, S., Liu, J., Yao, Y., Li, K., Yang, Y., 2007. The neural mechanism of human numerical inductive reasoning process: a combined ERP and fMRI study. In: Zhong, N., Liu, J.M., Yao, Y.Y., Wu, J.L., Lu, S.F., Li, K.C. (Eds.), Web Intelligence Meets Brain Informatics, pp. 223–243.

Libertus, M.E., Brannon, E.M., Pelphrey, K.A., 2009. Developmental changes in category-specific brain responses to numbers and letters in a working memory task. Neuroimage 44, 1404–1414.

Liu, J., Zhang, H., Chen, C., Chen, H., Cui, J., Zhou, X., 2017. The neural circuits for arithmetic principles. Neuroimage 147, 432–446.

Logie, R.H., Gilhooly, K.J., Wynn, V., 1994. Counting on working memory in arithmetic problem solving. Mem. Cognit. 22, 395–410.

Mazoyer, B., Zago, L., Mellet, E., Bricogne, S., Etard, O., Houdé, O., Crivello, F., Joliot, M., Petit, L., Tzourio-Mazoyer, N., 2001. Cortical networks for working memory and executive functions sustain the conscious resting state in man. Brain Res. Bull. 54, 287–298.

McKiernan, K.A., Kaufman, J.N., Kucera-Thompson, J., Binder, J.R., 2003. A parametric manipulation of factors affecting task-induced deactivation in functional neuroimaging. J. Cogn. Neurosci. 15, 394–408.

Menon, V., Rivera, S.M., White, C.D., Glover, G.H., Reiss, A.L., 2000. Dissociating prefrontal and parietal cortex activation during arithmetic processing. Neuroimage 12, 357–365.

Monti, M.M., Parsons, L.M., Osherson, D.N., 2012. Thought beyond language: neural dissociation of algebra and natural language. Psychol. Sci. 23, 914–922.

Nieder, A., 2004. The number domain- can we count on parietal cortex? Neuron 44, 407–409.

Niemi, D., 1996. Assessing conceptual understanding in mathematics: representations, problem solutions, justifications, and explanations. J. Educ. Res. 89, 351–363.

Noël, M.P., Désert, M., Aubrun, A., Seron, X., 2001. Involvement of short-term memory in complex mental calculation. Mem. Cognit. 29, 34–42.

Nunes, T., Bryant, P., Evans, D., Bell, D., Barros, R., 2012. Teaching children how to include the inversion principle in their reasoning about quantitative relations. Educ. Stud. Math. 79, 371–388.

Nunes, T., Bryant, P., Watson, A., 2009. Key Understandings in Mathematics Learning.

Passolunghi, M.C., Pazzaglia, F., 2004. Individual differences in memory updating in relation to arithmetic problem solving. Learn. Individ. Differ. 14, 219–230.

Passolunghi, M.C., Siegel, L.S., 2001. Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. J. Exp. Child. Psychol. 80, 44–57.

Piazza, M., Pinel, P., Le Bihan, D., Dehaene, S., 2007. A magnitude code common to numerosities and number symbols in human intraparietal cortex. Neuron 53, 293–305.

Poldrack, R.A., Wagner, A.D., Prull, M.W., Desmond, J.E., Glover, G.H., Gabrieli, J.D.E., 1999. Functional specialization for semantic and phonological processing in the left inferior prefrontal cortex. Neuroimage 10, 15–35.

Powell, S.R., Fuchs, L.S., 2014. Does early algebraic reasoning differ as a function of students' difficulty with calculations versus word problems? Learn. Disabil. Res. Pract. 29, 106–116.

Prabhakaran, V., Rypma, B., Gabrieli, J.D.E., 2001. Neural substrates of mathematical reasoning: a functional magnetic resonance imaging study of neocortical activation during performance of the necessary arithmetic operations test. Neuropsychology 15, 115–127.

Prado, J., Mutreja, R., Zhang, H., Mehta, R., Desroches, A.S., Minas, J.E., Booth, J.R., 2011. Distinct representations of subtraction and multiplication in the neural systems for numerosity and language. Hum. Brain Mapp. 32, 1932–1947.

Price, C.J., Moore, C.J., Humphreys, G.W., Wise, R.J.S., 1997. Segregating semantic from phonological processes during reading. J. Cogn. Neurosci. 9, 727–733.

Raichle, M.E., MacLeod, A.M., Snyder, A.Z., Powers, W.J., Gusnard, D.A., Shulman, G.L., 2001. A default mode of brain function. Proc. Natl. Acad. Sci. U. S. A. 98, 676–682.

Riley, M.S., Greeno, J.G., 1988. Developmental analysis of understanding language about quantities and of solving problems. Cognit. Instr. 5, 49–101.

Salillas, E., Semenza, C., Basso, D., Vecchi, T., Siegal, M., 2012. Single pulse TMS induced disruption to right and left parietal cortex on addition and multiplication. Neuroimage 59, 3159–3165.

Sandrini, M., Rossini, P.M., Miniussi, C., 2004. The differential involvement of inferior parietal lobule in number comparison: a rTMS study. Neuropsychologia 42, 1902–1909.

Semenza, C., Salillas, E., De Pallegrin, S., Della Puppa, A., 2016. Balancing the 2 hemispheres in simple calculation: evidence from direct cortical electrostimulation. Cereb. Cortex 2016, 1–9.

Shulman, G.L., Corbetta, M., Buckner, R.L., Fiez, J.A., Miezin, F.M., Raichle, M.E., Petersen, S.E., 1997. Common blood flow changes across visual tasks: I. Increases in subcortical structures and cerebellum but not in nonvisual cortex. J. Cogn. Neurosci. 9, 624–647.

Sohn, M.H., Goode, A., Koedinger, K.R., Stenger, V.A., Fissell, K., Carter, C.S., Anderson, J.R., 2004. Behavioral equivalence, but not neural equivalence–neural evidence of alternative strategies in mathematical thinking. Nat. Neurosci. 7, 1193–1194.

Swanson, H.L., Sachse-Lee, C., 2001. Mathematical problem solving and working memory in children with learning disabilities: both executive and phonological processes are important. J. Exp. Child. Psychol. 79, 294–321.

Thioux, M., Pesenti, M., Costes, N., De Volder, A., Seron, X., 2005. Task-independent semantic activation for numbers and animals. Brain Res. Cogn. Brain Res. 24, 284–290.

Tohgi, H., Saitoh, K., Takahashi, S., Takahashi, H., Utsugisawa, K., Yonezawa, H., 1995. Agraphia and acalculia after a left prefrontal (f1, f2) infarction. J. Neurol. Neurosur. Ps. 58, 629–632.

Tregellas, J.R., Davalos, D.B., Rojas, D.C., 2006. Effect of task difficulty on the functional anatomy of temporal processing. Neuroimage 32, 307–315.

van Harskamp, N.J., Rudge, P., Cipolotti, L., 2002. Are multiplication facts implemented by the left supramarginal and angular gyri? Neuropsychologia 40, 1786–1793.

Vukovic, R.K., Lesaux, N.K., 2013. The language of mathematics: investigating the ways language counts for children's mathematical development. J. Exp. Child. Psychol 115, 227–244.

Wei, W., Lu, H., Zhao, H., Chen, C., Dong, Q., Zhou, X., 2012a. Gender differences in children's arithmetic performance are accounted for by gender differences in language abilities. Psychol. Sci. 23, 320–330.

Wei, W., Yuan, H., Chen, C., Zhou, X., 2012b. Cognitive correlates of performance in advanced mathematics. Br. J. Educ. Psychol. 82, 157–181.

Woodcock, R.W., McGrew, K.S., Mather, N., 2001. Woodcock R, Mcgrew K S, Mather N. Woodcock-Johnson-III Tests of Achievement. Johnson.

Wu, T.H., Chen, C.L., Huang, Y.H., Liu, R.S., Hsieh, J.C., Lee, J.J., 2009. Effects of long-term practice and task complexity on brain activities when performing abacus-based mental calculations: a PET study. Eur. J. Nucl. Med. Mol. Imaging 36, 436–445.

Yang, Y., Liang, P., Lu, S., Li, K., Zhong, N., 2009. The role of the DLPFC in inductive reasoning of MCI patients and normal agings: an fMRI study. Sci. China. C. Life. Sci. 52, 789–795.

Yu, X., Chen, C., Pu, S., Wu, C., Li, Y., Jiang, T., Zhou, X., 2011. Dissociation of subtraction and multiplication in the right parietal cortex: evidence from intraoperative cortical electrostimulation. Neuropsychologia 49, 2889–2895.

Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B., Tzourio-Mazoyer, N., 2001. Neural correlates of simple and complex mental calculation. Neuroimage 13, 314–327.

Zhang, H., Chen, C., Zhou, X., 2012. Neural correlates of numbers and mathematical terms. Neuroimage 60, 230–240.

Zhang, Y., Chen, C., Liu, H., Cui, J., Zhou, X., 2016. Both non-symbolic and symbolic quantity processing are important for arithmetical computation but not for mathematical reasoning. J. Cogn. Psychol. 28, 807–824.

Zheng, X., Swanson, H.L., Marcoulides, G.A., 2011. Working memory components as predictors of children's mathematical word problem solving. J. Exp. Child. Psychol. 110, 481–498.

Zhong, N., 2008. Unifying study on human and web problem solving: a brain informatics perspective. IEEE Grc 2008, 25–26.

Zhou, X., Chen, C., Dong, Q., Zhang, H., Zhou, R., Zhao, H., Chen, C., Qiao, S., Jiang, T., Guo, Y., 2006. Event-related potentials of single-digit addition, subtraction, and multiplication. Neuropsychologia 44, 2500–2507.

Zhou, X., Chen, C., Qiao, S., Chen, C., Chen, L., Lu, N., Dong, Q., 2009. Event-related potentials for simple arithmetic in Arabic digits and Chinese number words: a study of the mental representation of arithmetic facts through notation and operation effects. Brain Res. 1302, 212–224.

Zhou, X., Chen, C., Zang, Y., Dong, Q., Chen, C., Qiao, S., Gong, Q., 2007. Dissociated brain organization for single-digit addition and multiplication. Neuroimage 35, 871–880.