Interactive Learning Environments

ISSN: 1049-4820 (Print) 1744-5191 (Online) Journal homepage: https://www.tandfonline.com/loi/nile20

Interaction patterns in exploratory learning environments for mathematics: a sequential analysis of feedback and external representations in Chinese schools

Jingjing Zhang, Ming Gao, Wayne Holmes, Manolis Mavrikis & Ning Ma

INTERACTIVE LEARNING ENVIRONMENTS
https://doi.org/10.1080/10494820.2019.1620290

Interaction patterns in exploratory learning environments for mathematics: a sequential analysis of feedback and external representations in Chinese schools

Jingjing Zhang [a], Ming Gao [b], Wayne Holmes [c], Manolis Mavrikis [d] and Ning Ma[e]

[a]Big Data Centre for Technology-mediated Education, Beijing Normal University, Beijing, People's Republic of China; [b]Research Centre of Distance Education, Beijing Normal University, Beijing, People's Republic of China; [c]Institute of Educational Technology, Open University, Milton Kin, UK; [d]UCL Knowledge Lab, University College London, London, UK; [e]Advanced Innovation Center for Future Education, Beijing Normal University, Beijing, People's Republic of China

ABSTRACT
Feedback in exploratory learning systems has been depicted as an important contributor to encourage exploration. However, few studies have explored learners' interaction patterns associated with feedback and the use of external representations in exploratory learning environments. This study used Fractions Lab, an exploratory learning environment for mathematics, to facilitate children's conceptual understanding of fractions in three Chinese schools. Students ($n = 189$) from six different classes were invited to use Fractions Lab, and 260,000 event logs were collected. Beyond demonstrating the overall efficacy of the approach, lag sequential analysis supported us in approaching a deeper understanding of patterns of interaction. The findings highlight that the design of three-levels of feedback (Socratic, guidance, and didactic-procedural feedback) played different roles in supporting students to use external representations to perform mathematical tasks in an exploratory learning environment. This study sheds light on how these interaction patterns might be applied to the Fractions Lab system in order to provide increasingly tailored support, based on cultural differences, to enhance students' technology-mediated learning experiences.

ARTICLE HISTORY
Received 31 October 2018
Accepted 14 May 2019

KEYWORDS
ITS; feedback; external representation; exploratory learning; maths education

Introduction

Research in the learning sciences emphasises the important role of feedback in using external representations to aid reasoning and problem solving (Johnson, Reisslein, & Reisslein, 2015). Of particular interest in this age of artificial intelligence, is the application of feedback to exploratory or discovery learning that is supported by exploratory learning systems (ELEs). To be most effective in ELEs, students usually require some form of support, such as feedback, scaffolding, or elicited explanations, to aid their use of external representations (such as diagrams). This need has been established in various domains, including analogical reasoning, vector arithmetic, algebra word problems, and logical and analytical reasoning (e.g. Ainsworth, Bibby, & Wood, 2002; Cox, 1997; Larkin & Simon, 1987; Stenning, 2002).

Although Shanghai (China) topped the PISA 2015 list, mathematics is not taught through an exploratory approach in China. With an emphasis on "mastery learning", direct instruction has been identified as the most effective teaching approach for success in final examinations (Boylan et al., 2016). This is one of the important reasons why instructive feedback is used widely in

CONTACT Ning Ma horsening@bnu.edu.cn; Ming Gao mgao519@mail.bnu.edu.cn

China's classrooms, while reflective prompts, affect boosts, and affirmation prompts are not frequently used. The core mathematics topics are first taught by teachers and then practised rigidly by students so that they master the topics rather than being involved in self-directed inquiry (An, Kulm, & Wu, 2004). Mathematics teaching in the Chinese context is still traditional in nature, as the Confucian culture regards the teacher as the source of knowledge (Zhang, 2009). Due to the fact that teachers dominate the delivery of knowledge, which means an unequal relationship between teachers and students, explorative learning is rarely carried out in Chinese classrooms. With such teacher-led practices, teachers provide instructive feedback to help their students to understand or complete quizzes that are prepared before class (Cai & Wang, 2010). Teachers tend to teach students how to solve problems directly, using verbal communication (Li, Cao, & Mok, 2016), or by telling them what is wrong when they cannot identify their mistakes. Chinese students are under high pressure to compete in examinations and therefore exploratory learning, which can require more time than the traditional approach (i.e. direct instruction) (Kong, 2008), is rarely prioritised. However, although direct instruction can help students to solve similar problems, such approaches do not help them to develop more general mathematical thinking skills that can be used to solve real-world problems (Kapur, 2014).

Rather than focusing on mathematics problem solving, mathematics education in China emphasises the practice of arithmetical computation (Zhou et al., 2018). As the goal of mathematics education is not to develop student competencies to solve real-world problems, there is also a lack of emphasis on the role of external representations. In schools, students are trained to solve mathematical problems by mimicking what they are told in class. Tasks are repeated after class to develop the quick triggering of internal representations to help answer questions. For example, fractions, a core mathematics topic that can prove to be difficult for young students to master, is taught by developing procedural and conceptual knowledge through traditional, more rigid practices, instead of by designing activities to promote creativity and inquiry to develop students' understanding of the fraction concepts (An et al., 2004). Although Suthers (2001) showed how the choice of representation can influence an individual's conception of a problem, and how it triggers an internal representation (and therefore makes it easier to find a solution to the problem), external representations are not adopted widely in the Chinese culture, which is oriented towards score-based learning. This is perhaps due to the fact that the mathematics is viewed as the absolute truth, hence it is thought to be unnecessary to use external representations to facilitate discussing or reflecting upon the truth of knowledge (Zhang, 2009). Direct instruction, direct memorising, and drills and practice, rather than mathematical problem solving, are designed to enhance arithmetical computation (Zhou et al., 2018). In fact, it has been argued that mathematics education in China has, unfortunately, failed to develop students' passion and enthusiasm towards the subject. Research has also shown that, although Chinese students perform better on exams or in PISA than their counterparts elsewhere in the world, they lack creative problem-solving skills to solve real-world problems (Cai, 2000; Cai & Silver, 1995; Wang & Lin, 2005).

Accordingly, the collaborative research that is reported in this paper has introduced Fractions Lab to Chinese schools. Fractions Lab is a virtual manipulative ELE (Hansen, Mavrikis, & Geraniou, 2016) with exploratory tasks and intelligent feedback. Students are provided with a range of different external representations that they can manipulate to solve fraction problems. The current version of Fractions Lab provides a range of feedback to students (including instructive feedback, reflective prompts, affect boosts, and affirmation prompts). The study builds on earlier research in the UK and Germany (Rummel et al., 2016) by investigating the use of Fractions Lab in three schools in Beijing, China, with 189 children. In particular, beyond learning gains, we examined the learning trajectories associated with feedback interactions and the use of external representations. In China, it remains unclear how students' interactions with the exploratory learning environment through the use of mathematical representations is linked to feedback, and how this kind of information can be used to improve this environment. Thus, this research attempted to answer the following three research questions: (1) How do learners interact with Fractions Lab? (2) Are there any patterns in the interactions and do learners engage in certain interaction sequences repeatedly? (3) Which kinds of external representations dominate in explorative tasks after learners have received feedback? Understanding the role of multiple external representations and intelligent feedback in students' learning trajectories has the potential to provide increasingly tailored support based on individual student behaviours, to improve students' learning experiences.

The strengths and challenges of exploratory learning environments

Exploratory Learning Environments are used to support exploratory or discovery learning, with some offering learners multiple representations that help them identify their own routes to achieving the learning goals. External representations (such as diagrams) can be powerful aids to reasoning and problem-solving. For example, Zhang (2002) described how external representations are able to guide, constrain, and determine cognitive behaviour. ELEs have also been used in multiple disciplines to support students' knowledge construction, especially in science education, which is typically exploratory in nature. For example, Chang, Chen, Lin, and Sung (2008) found that students who used an exploratory learning simulation to learn the abstract reasoning abilities of physics performed better than those who did not use the simultion. Pyatt and Sims (2012) compared the attitudes towards, performance in, and access to inquiry investigation that occurs in inquiry-based (i.e. exploratory) science labs in first-year secondary school chemistry classes. These authors reported that the students showed a preference for the inquiry-based learning environments.

Research has also shown that learning in such environments raises challenges associated with the cognitive and metacognitive complexities of the learning experience that these environments offer (Azevedo, 2005; Scheiter & Gerjets, 2007). Kirschner, Sweller, and Clark (2006) reported that unguided open-ended discovery might lead to unproductive floundering, and thus be less effective than direct instruction. According to Davis (2000), one way to help students overcome these challenges is by providing some form of support, such as feedback, scaffolding, or elicited explanations. Feedback, in particular, is thought to be especially important. However, while feedback has been researched extensively over the decades (e.g. Aleven, Stahl, Schworm, Fischer, & Wallace, 2003), further work is needed to operationalise it in exploratory learning environments. For example, as suggested by Carenini et al. (2014), any systematic approach to operationalising feedback in an exploratory learning environment needs to address the "when", "what", and "how" questions: when it should be provided; what it should contain; and how it should be presented. A range of different kinds of feedback (including instructive feedback, reflective prompts, affect boosts, and affirmation prompts) can maximise the potential of ELEs to foster enhanced learning experience in schools (Grawemeyer et al., 2016; Holmes, Mavrikis, Hansen, & Grawemeyer, 2015). As the levels of learners' precognition differ, feedback should be tailored to individuals, instead of providing exactly the same learning experiences for everyone (Nazemi, Breyer, Stab, Burkhardt, & Fellner, 2010). To achieve this, artificial intelligence techniques drawn from Intelligent Tutoring Systems (ITSs) (Mousavinasab et al., 2018) have sometimes been employed (e.g. Rummel et al., 2016). In addition, researchers have realised that it is important to shift from measuring the learning outcomes of, in the case of this study, fraction knowledge, to interpreting the learning process where learning occurs (Kong, 2008; Mavrikis, Holmes, Zhang, & Ma, 2018).

Learning analytics and lag sequential analysis

The widespread use of ITSs and ELEs generate increasingly large set of data, including interaction data, personal data, and academic achievements (Romero, Ventura, & García, 2008). The built-in analytics are often basic or non-existent in these systems, incapable of enhancing the depth of extraction aggregation, reporting and visualisation (Dawson, 2010). The fields of learning analytics (LA) and educational data mining (EDM) have been challenged by how to take value out of these big sets of learning-related data to optimise opportunities for learning (Ferguson, 2012) In particularly, as Siemens

(2013) proposed, it is important to push learning analytics as a discipline to better understand teaching, learning, "intelligent content", and personalisation and adaption. This perspective particularly applies to measure and analyse the pre-existing and machine-readable data stored in ITSs and ELEs, where teaching and learning are facilitated via "intelligent content".

Technical approaches to LA/EDM can be classified in this way include: prediction, clustering, relationship mining, distillation of data for human judgment, and discovery with models (Baker & Yacef, 2009). Also, Bienkowski, Feng, and Means (2012) proposed five areas of LA/EDM application: modelling user knowledge, behaviour, and experience; creating profiles of users; modelling knowledge domains; Trend analysis; personalisation and adaptation. One particular approach, lag sequential analysis (LSA), is situated within the technical domain of discovery with models, and the application area of modelling user knowledge and experience. A significant development has been the integration of LSA within the temporality perspective of learning analytics, as the temporal nature of learning (hidden within high-resolution temporal data) is arguably situated in the central of learning analytics, which is previously underexamined (Knight, Wise, & Chen, 2017).

The use of LSA, a method proposed by Sackett (1978), allows researchers to carry out detailed investigation of learning behaviours or event chains that occur at frequencies greater than chance. In the educational domain, LSA takes transitional relationships into consideration to identify temporal differences in learning behaviours (Chen, Resendes, Chai, & Hong, 2017). In fact, the use of LSA (e.g. Gunter, Jack, Shores, Carrell, & Flowers, 1993; Jeong, 2003; Putnam, 1983) predates the emergence of LA and EDM. From 2013, LSA has been used for a variety of purposes: to identify common patterns that potentially help instructors provide personalised feedback as needed, which involves identifying patterns of student learning behaviour transitions (Hou, 2012; Hwang & Chen, 2017; Yin et al., 2017); exploring the content and patterns of teacher discussions (Hou, Sung, & Chang, 2009); exploring interaction patterns during knowledge construction (Lan, Tsai, Yang, & Hung, 2012; Lin, Duh, Li, Wang, & Tsai, 2013; Yang, Li, Guo, & Li, 2015); and investigating the learning behaviours of high-achievement or low-achievement students or confirming the relationship between student interaction transitions and learning outcomes (Cheng, Wang, Cheng, & Chen, 2019; Lai & Hwang, 2015; Yang, Guo, & Yu, 2016).

Fractions lab

Fractions Lab is an exploratory learning environment that is intended to facilitate children's conceptual understanding of fractions via the direct manipulation of representations. It originated in the EU-funded iTalk2Learn project's intelligent tutoring platform that aims to support mathematics learning for children aged 5–11 years. The overall efficacy of the platform has been verified through instructional evidence from the UK, Germany, and China (Grawemeyer et al., 2017; Mavrikis et al., 2018; Rummel et al., 2016).

With the aim of fostering conceptual knowledge, some external representations, affordances (tools), and constraints were designed (Hansen et al., 2016). As shown in Figure 1(a), seven different external representations, i.e. liquid in a beaker shape (Liquid), Number Line (Line), Horizontal Rectangle (Hrect), Vertical Rectangle (Vrect), a Set of Stars (Starset), a Set of Moons (Moonset), and a Set of Hearts (Heartset), can be selected to perform mathematical tasks in part A. Students can also use tools (indicated in part B) to check their hypotheses and adopt a constructivist stance to learning. The trashcan shown in part C allows representations to be deleted. Three different levels of feedback were designed, to be flexible and to help students to learn from their errors (Holmes et al., 2015). The feedback is delivered automatically in response to the students' interactions, or it can appear when the student clicks on the question mark shown in part D (Grawemeyer et al., 2017). The first level of feedback, Socratic feedback, encourages students to verbalise possible solutions (such as "*What do you need to do to the denominators and numerators?*"). The second level, guidance, reminds the students of key domain-specific rules and the system's affordances (such as "*To compare the fractions easily, the denominators of each fraction need to be the same*"). The third level, didactic-procedural,

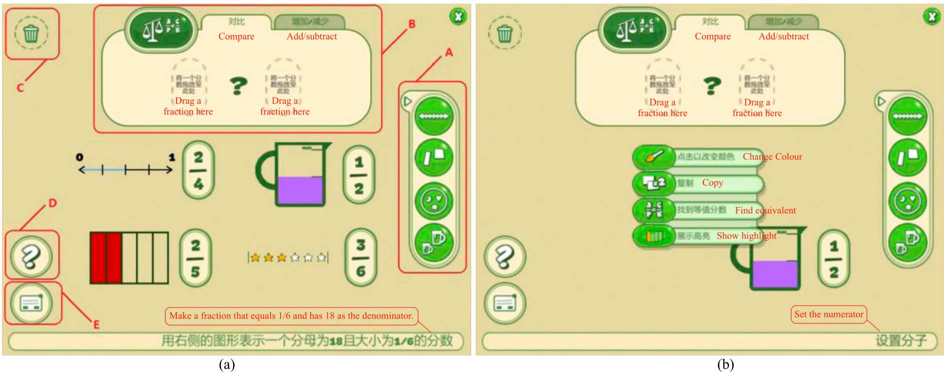

Figure 1. Screenshots of the Fractions Lab interface.

specifies a possible next step in terms of the fraction concepts that are currently being explored (such as "*Multiply the top and bottom of one fraction with the bottom of the other fraction. Do the same with the other fraction. Then you can compare the fractions*".) (Holmes et al., 2015). The description of the present task and the manner of resetting can be attained by clicking on the area marked E. Some affordances or tools provide functions to copy the fraction, find the equivalent fraction, or highlight some parts, as shown in Figure 1(b).

Methodology

Participants and data collection

The study involved three primary/elementary schools (inner-city and suburban contexts) that were all in Beijing. One-hundred-and-eighty-nine students (aged between 9 and 10 years) participated. All of them were new to Fractions Lab. In each class, the students engaged with it for approximately 45 min to complete the 18 provided tasks. The tasks were grouped into four: creation, comparison, addition, and subtraction.

As the students interacted with the system, Fractions Lab automatically generated a comprehensive range of data (e.g. student interaction data and data about feedback), all of which were saved in a database with unique student IDs and time stamps. The interaction data included the mathematical task on which the student was currently working, the representation that the student selected to perform the mathematical task, and the fraction that the student generated or changed (e.g. change the numerator or denominator of a fraction). The feedback data included when and what type of feedback was provided to the student. As few students managed to perform the last two tasks, only the event logs generated from 16 tasks were included in the data analysis. In total, approximately 260,000 event logs were collected. We also recorded the affect states and task difficulty using self-reports.

The pre- and post-tests were developed by two experts who had more than 10 years' experience of teaching mathematics and were validated by Chinese mathematics teachers. The tests were designed to assess the students' prior and subsequent procedural and conceptual knowledge of fractions. Both tests were comprised of six single-choice items, with one point for each item.

Data cleaning and categorisation

As all participants were new to Fractions Lab, they were guided by teachers and used Task 1 to explore the environment. Thus, for our data analysis, Task 1 was removed. The data for non-productive repeated behaviours (such as repeatedly changing the denominator or numerator) were also removed. In total, approximately 46,000 event logs were used for the sequential analysis. The

Table 1. Activity categories and associated events.

Code	Activity	Contains event
GenF	Generating fraction	FractionGenerated (contains using different external representations)
ChaF	Changing the denominator or numerator of a fraction	FractionChange (contains different external representations)
TraF	Deleting the fraction	FractionTrashed, EquivalenceTrashed, CutTrashed,
LabC	Dragging fractions to balance to compare, add, or subtract	OperationResult, FractionPlaced, ClickButton_OperationSwitchMinus, ClickButton_3OperandTab, ClickButton_2OperandTab, ClickButton_OperationSwitchPlus
TasO	Opening the description of the present task	ClickButton_OpenTaskDescription
TasR	Resetting the present task	TaskReset
SeeS	Seeking scaffolding to solve the problem, such as creating the equivalent fraction, changing the colour of the numerator part, etc.	EquivalenceGenerated, ChangeColour, HighlightFraction, DehighlightFraction, SystemHighlight
StaR	Completing the task (true, false, or unfinished)	EndStateReached, TaskChange_warningMessage,
FeeB	Requesting feedback to resolve the task	Feedback (contains three levels)

behaviour logs were grouped into nine activity categories based on the types of interactions, as shown in Table 1.

Data analysis

To gain an understanding of how the participants used Fractions Lab, descriptive analyses of their learning behaviours (e.g. the number and levels of feedback requested, the different types of external representations generated) were carried out. A one-way analysis of variance (ANOVA) was used to compare the differences among the three schools in the pre- and post-tests, and paired samples t-tests were used to analyse student learning gains.

As shown in Table 1, the event logs were grouped into nine activities according to the design of Fractions Lab. The transitional probabilities, which represent the likelihood of moving from one activity to another or staying with the same activity, were calculated. An extensive applied lag sequential analysis method (Sackett, 1978) was used to identify activity chains that occurred at frequencies greater than chance. As is the usual practice, the z-score, which is attained from the adjusted residual of each transition (Bakeman & Gottman, 1997), was used to estimate the statistical significance of a sequence. That is, if the adjusted residuals value was greater than 1.96, then it was significant at the level of 0.05. However, it is also important to point out that a single z-score is insufficient to indicate the degree to which a pattern is present (McComas et al., 2009; Wampold, 1992). Yule's Q, a strengthening association measurement, has been suggested for use in combination with the z-score. This is a transformation of the odds ratio in the range of −1 to +1. A sequence can be seen as having a moderate association when its Q-value is at least 0.3 (Davis, 1971). Thus, we associated the z-value and Q-value to verify the significant sequence. Version 5.1 of the generalised sequential querier (GSEQ 5.1), which contains the above two measurements, was adopted to analyse the activity sequences of the students in our study.

Results

Learning experiences in fractions lab

The three different types of tasks were used to investigate the students' perceptions of task difficulty (using the self-reports completed between each task). The data showed that the level of difficulty was perceived as moderate, which implies that the students were capable of undertaking the learning tasks with the support of feedback in this exploratory learning environment.

There was a general tendency towards help-avoidance, which is consistent with previous studies (Mavrikis, Grawemeyer, Hansen, & Gutierrez-Santos, 2014; Roll, Baker, Aleven, & Koedinger, 2014).

Table 2. Description of the external representations generated.

School	Class	No. of external representations created	Top three types of external representations	No. (%) of external representations
School A	Class A1	668	Liquid	322 (48.2%)
			Number line	207 (31.0%)
			Heartset	44 (6.6%)
	Class A2	566	Vertical rectangle	168 (29.7%)
			Liquid	142 (25.1%)
			Number line	112 (19.8%)
School B	Class B1	830	Liquid	228 (27.5%)
			Vertical rectangle	223 (26.9%)
			Number line	187 (22.5%)
	Class B2	975	Liquid	395 (40.5%)
			Number line	225 (23.1%)
			Vertical rectangle	163 (16.7%)
School C	Class C1	1026	Vertical rectangle	446 (43.5%)
			Liquid	252 (24.6%)
			Horizontal Rectangle	98 (9.6%)
	Class C2	830	Liquid	270 (32.5%)
			Vertical rectangle	185 (22.3%)
			Number line	111 (13.4%)

Moreover, although we did not observe extensive gaming, perhaps due to the novelty effects and the engaging nature of the exploratory tasks, a paired t-test showed statistically significant differences (a large effect size and a Cohen's d of 1.85) in the post-test grades of a group of students who attempted to abandon the tasks without spending time interacting ($M = 2.935$, SD = 0.245), when compared with a group who did not attempt to game the system ($M = 3.287$, SD = 0.109; t (195) = 1.292, $P < 0.05$).

The most frequently used external representations were Liquid, Vertical rectangle, Number line, Horizontal rectangle, and Heartset (see Table 2). Although seven different external representations are provided in Fractions Lab, the students tended to use the "Liquid" representation (liquid in a beaker shape) most frequently. This suggests that the students found the "Liquid" representation most helpful to understand the meaning of fractions and to see the relationship between value and volume.

Learning gains in fractions lab

A one-way ANOVA was used to analyse the differences among the three schools' pre-test and post-test scores. The results show that there was a significant difference among the three schools in the pre-test scores ($F(2, 194) = 11.294$, $p < 0.001$), and there was no significant difference among the three schools in the post-test scores ($F(2, 194) = 1.828$, $p > 0.05$). Further analysis showed that there was a statistically significant difference between School C (in a suburb of Beijing) and the other two schools ($p < 0.001$), but no significant difference was found between School A and School B, both located in the inner city ($p = 0.638$). The students from School C (in a suburb of Beijing) scored the lowest in the pre-test ($M = 1.69$, SD = 1.207), while School A ($M = 2.53$, SD = 1.268) and School B scored relatively higher ($M = 2.63$, SD = 1.180). Paired samples t-tests show statistically significant differences between the pre- and post-tests (Table 3) for all three schools, with effect sizes (Cohen's d) of 0.44 (School A), 0.70 (School B), and 1.00 (School C). As shown in Table 3, students with lower ability (who scored low in the pre-test) from the suburb appear to have gained more by using Fractions Lab ($M = 3.11$, SD = 1.575, $t = 7.102$, df = 63, p <0.05).

Table 3. Pre- and post-test scores (each out of a possible 6) of students in Chinese schools.

School	n	Pre-test	Post-test	t-test
School A	64	$M = 2.53$, SD = 1.268	$M = 3.11$, SD = 1.326	T (65) = 3.470, $p = 0.01$
School B	63	$M = 2.63$, SD = 1.180	$M = 3.48$, SD = 1.239	T (64) = 4.308, $p < 0.05$
School C	62	$M = 1.69$, SD = 1.207	$M = 3.11$, SD = 1.575	T (63) = 7.102, $p < 0.05$

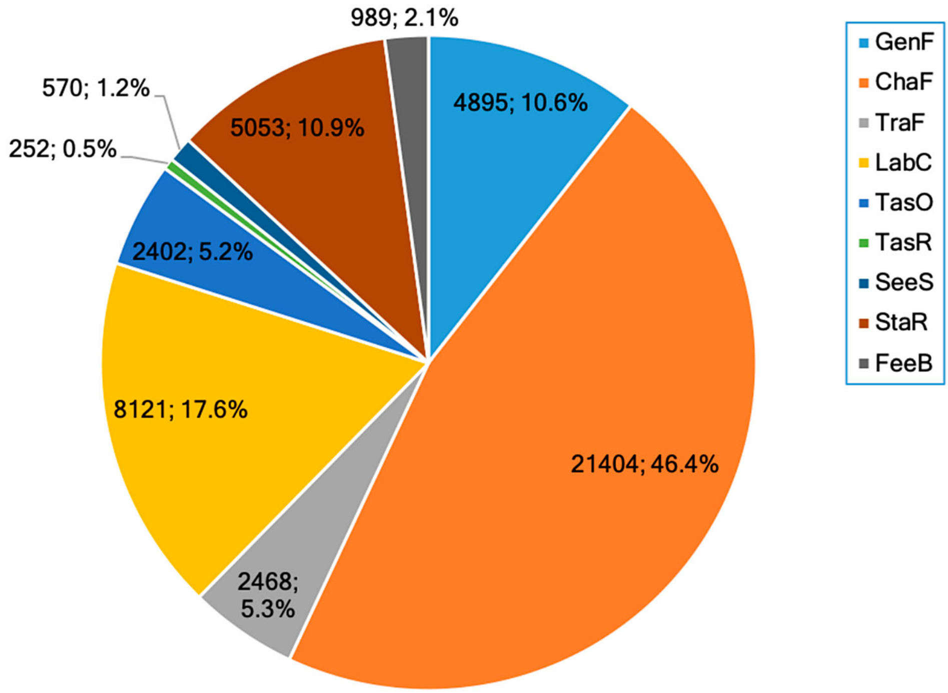

Figure 2. Frequency distribution of the nine learning activities.

How did students interact with fractions lab?

As shown in Figure 2, the frequencies of the students' interaction in nine activities differed greatly.

The most common activities were changing the denominator or numerator of a fraction (ChaF, 46.4%, 21,404) and dragging fractions to the balance tool in order to compare, add, or subtract (LabC, 17.6%, 8121). The results indicated that students frequently used the function of the "lab" area (indicated in part B of Figure 1(a)) to help them verify hypotheses by comparing, adding, or subtracting fractions. After creating the fractions, the students tended to set their values first, to make another attempt, before requesting feedback. Activities, such as requesting feedback (FeeB, 2.1%, 989) and seeking scaffolding (SeeS, 1.2%, 569), were carried out less frequently. The students appeared to have explored more often than they asked for help.

Interaction patterns

There was a total of 81 activity transitions. The probabilities of transitions from one activity to another activity or remaining on the same activity are shown in Figure 3. The y-axis shows the activities that were initiated by students; the x-axis represents the activities that the students sequentially followed. The depth of colour represents the transitional probability value.

After creating a fraction, the students had a higher tendency to set the value of the fractions than to perform other activities (which is indicated by the darkest colour box in the row of GenF that crosses the ChaF column). After changing the fractions, the students were more likely to change their values again (ChaF), drag the fraction to the balance (LabC), and reach some state (StaR). This is illustrated in the darker colour box in the row of ChaF that crosses the ChaF, LabC, and StaR columns. After trashing the fractions and resetting the tasks, the students tended to create a new fraction, as illustrated in the dark colour box in the row of TraF and TasR that crosses the GenF column.

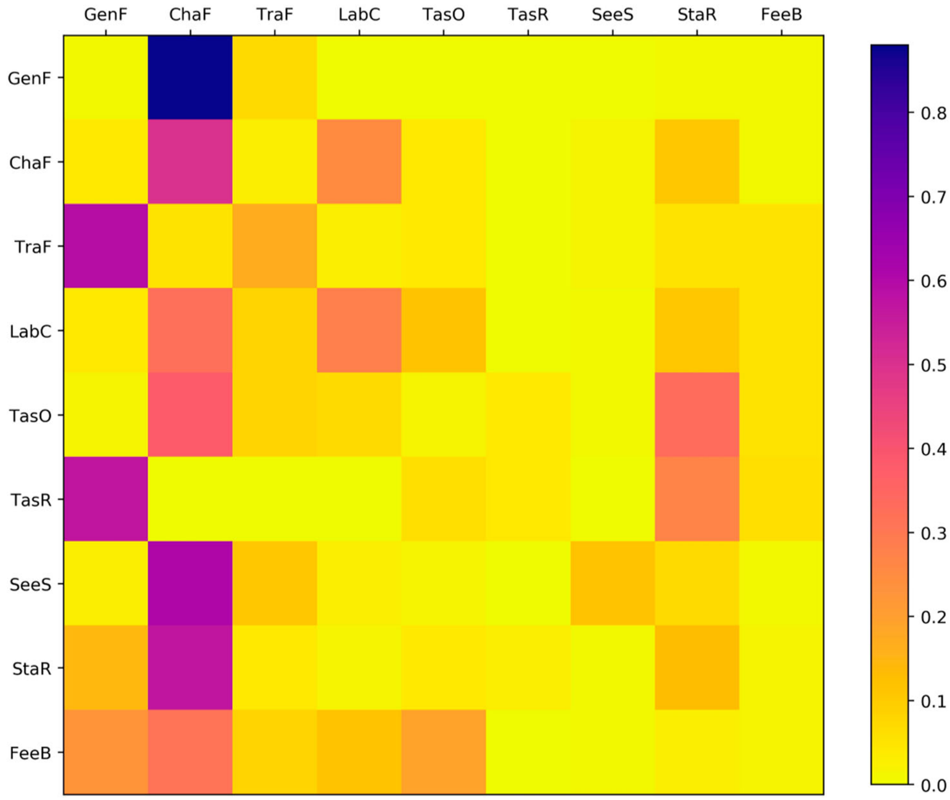

Figure 3. Transition probability between activities.

As mentioned earlier, the sequences were considered statistically significant only if the z-value was greater than 1.96 and the Q-value was at least 0.30. The lag sequential analysis shows that 21 activity sequences were statistically significant, as illustrated in Figure 4. The arrows represent the transitional direction from one activity to another (including itself). The statistically significant sequential relationship between *GenF* and *ChaF* indicates that the students typically set the value of a fraction immediately after creating it. In terms of using scaffolding (e.g. creating the equivalent fraction, changing the colour of the numerator part) to assist a task, *SeeS* had sequential relationships with *TraF* (trashing the fraction) and itself. To a large extent, the students tended to use different kinds of scaffolding to help them explore the learning tasks. After trying out different kinds of scaffolding repeatedly, they deleted the fraction and generated a new one.

The amount of learning activities that were statistically significantly associated with feedback was the largest, including both directional sequential relationships from feedback or to feedback (i.e. *FeeB→GenF*, *TasR→FeeB*, *TraF→FeeB*, *LabC→FeeB*) and bi-directional sequential relationships (i.e. *FeeB→TasO*, *TasO→FeeB*). After receiving feedback, the students were more likely to generate a new fraction with the same or different external representations to explore, or simply to go back to open the task descriptions for clarification.

Feedback, external representation generated, and fraction value changed

To explore more deeply the ways in which the use of different levels of feedback affected the use of different external representations, the *GenF* activity was further divided into seven sub-activities, in which each distinctive external representation was generated by students (GenFline, GenFhrect,

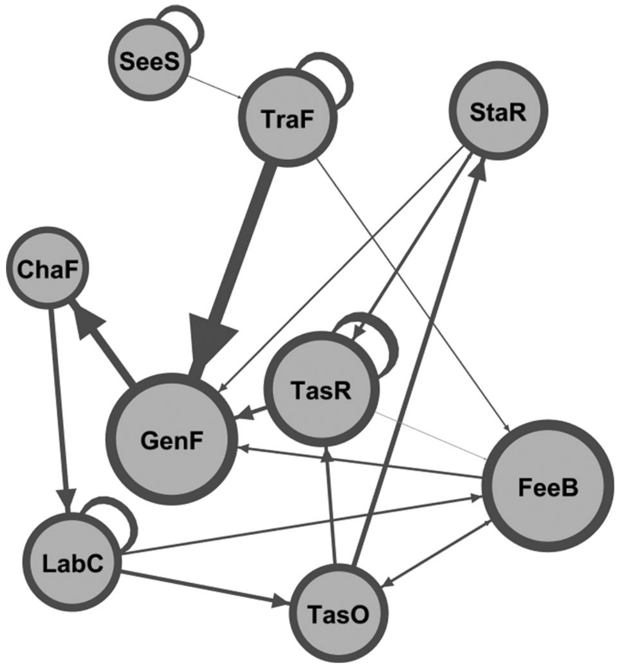

Figure 4. Activity transition diagram of significance (z-score > 1.96, $Q \geq 0.30$). The boldness of the links indicates their z-score.

GenFvrect, GenFstarsets, GenFheartsets, GenFmoonsets, and GenFliquid). By doing so, it is possible to examine how students manipulate these seven external representations (i.e. Liquid, Number Line, Horizontal Rectangle, Vertical Rectangle, a Set of Stars, a Set of Moons, and a Set of Hearts) to solve fraction problems. The *FeeB* was separated into three sub-activities regarding the levels of feedback requested (FeeBl1, FeeBl2, and FeeBl3); while the *ChaF* was divided into two sub-activities based on the external representation of the fraction used (*ChaFS*, changing the fraction by using the same external representation; and *ChaFO*, changing the fraction by using a different external representation). By examining these sub-categories of GenF and FeeB further in great detail, 11 new activity categories were generated. In total, 18 activities (including the original SeeS, TraF, StaR, TasR, LabC, and TasO) were analysed by lag sequential analysis to test the significance of the transitions among these activities.

As shown in Figure 5, the activity of changing the fraction by using the same external representation (ChaFS) usually occurred after the fraction was generated with all types of external representations. The sequential relationship that links ChaFS and the activity of dragging fractions to balance to compare, add or substract (LabC) indicates that the students tended to drag fractions to the balance to verify their hypothesis after the values of the fraction had been set. The activity of changing the fraction by using a different external representations (ChaFO) was commonly followed by the activity of seeking scaffoldings (SeeS), deleting the fraction (TraF), requesting the third-level feedback (FeeBl3), and ChaFO activities. In addition, the line boldness shows a higher sequential relationship within ChaFO (ChaFO ->ChaFO), and this indicates that the students frequently changed a fraction from one representation to a different one. Moreover, there was a tendency to seek scaffolding (SeeS) after they had changed the value of a different representation.

As shown in Figure 6, after receiving the first-level feedback, the students tended to generate a new fraction with different types of external representations or to open the task description for clarification. The first-level feedback (FeeBl1) was commonly requested after the students had explored Fractions Lab themselves by using the balance (LabC).

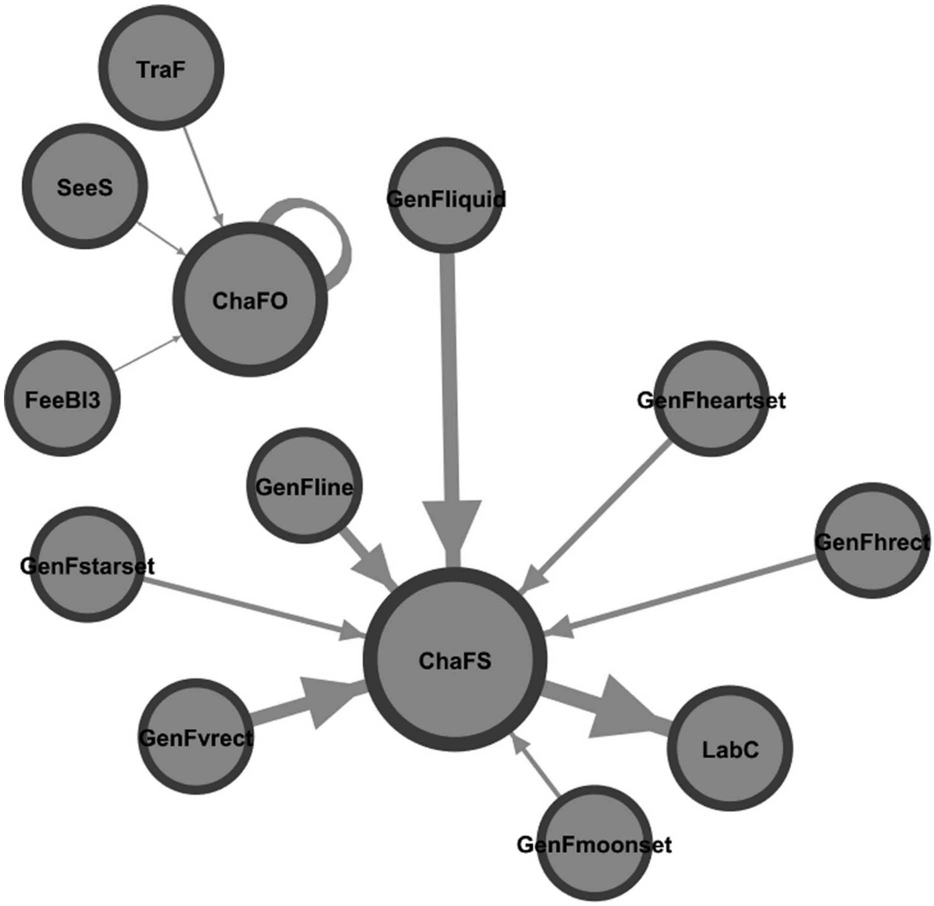

Figure 5. Significant transitions that relate to ChaF activities (the top shows the transitional relationships of ChaFO, and the bottom shows the transitional relationships of ChaFS).

In contrast to *FeeBl1*, after receiving the second-level feedback, the students tended to generate a new fraction to keep working. However, the most commonly used representation "Liquid" did not appear to be significant (See Figure 7). Second-level feedback (*FeeBl2*) was requested after opening the task description or trashing the fraction. Interestingly, the students tended to request the second-level feedback repeatedly; this did not occur commonly when they requested the first-level feedback.

As shown in Figure 8, there were fewer significant transitions between *FeeBl3* and other activities. Similar to the first level of feedback requests, the students were more likely to request the third-level feedback (*FeeBl3*) after using the balance. However, there were only two significant transitions after they received the third-level feedback. In contrast to the first-level and second-level of feedback requests, they tended to change the value of the representational fraction, instead of generating different types of external representations.

Discussion

This study has explored how Chinese students interacted with an exploratory learning environment through the use of mathematical representations and how this was linked to feedback interactions. The study showed that there was a significant transition from requesting feedback (FeeB) to generating

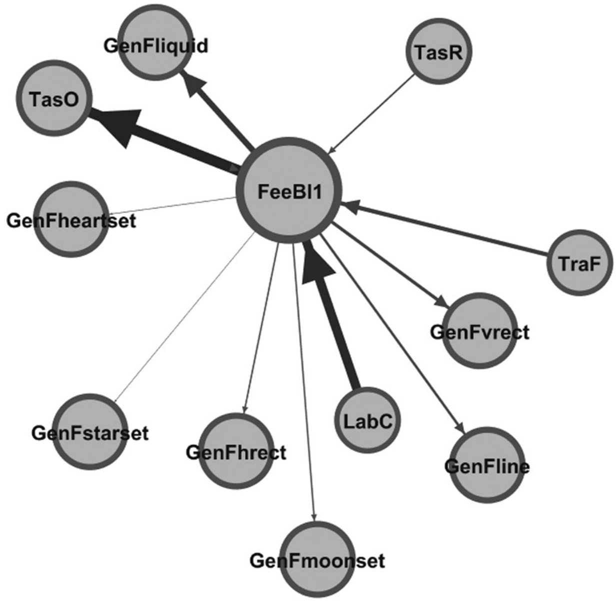

Figure 6. Significant transitions that relate to FeeBl1 activity.

fractions with representations (GenF), and this implies that feedback provided in Fractions Lab, to some extent, did guide students to use representations in order to learn. The findings highlight that the design of three-level feedback (i.e. Socratic, guidance, and didactic-procedural feedback) indeed played different roles in supporting students to perform mathematical tasks in an exploratory learning environment, which is consistent with the proposal stated in Holmes et al. (2015).

In Fractions Lab, the students did tend to use representations to explore and to complete the learning tasks, which is evidenced by the fact that the activities of seeking scaffoldings to solve the problem (SeeS) were commonly used before or after changing the fraction by using a different external presentation (ChaFO). The students also tended to explore different types of external representations to make sense of fractions, which is evident from a higher sequential relationship within ChaFO. In a culture where students are trained by classroom instruction to complete learning tasks efficiently in examinations to ensure high scores (Cai & Nie, 2007), it is encouraging to see that they slowed down the process of completing a task by engaging in some exploration using Fractions Lab. AI-supported ELEs, such as Fractions Lab, might function as open and flexible learning environments to allow students to explore, and in this way, further develop their problem-solving skills, which are unlikely to be developed in classroom instruction.

First-level feedback was usually requested after carrying out the activities of dragging fractions to balance to compare, add or subtract (LabC), and this further demonstrates that students need some form of support to complete their learning tasks in exploratory learning. Such significant transition highlights the importance of feedback, which is also emphasised in the literature (e.g. Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Holmes et al., 2015). It is interesting to point out that, before requesting the first-level feedback, the students were still in the exploratory mood (e.g. using LabC). After requesting the first-level feedback, they tended to generate different forms of external representations to explore, with the Liquid form seeming to be the most frequently used representation. Nevertheless, when they requested the second-level feedback, they rarely used LabC for exploration, and fewer external presentations were used. First-level (Socratic) feedback did not help them to complete the learning task, and it appears that second-level feedback (guidance feedback) might have lead them to be more confused because of the increased cognitive load (Grawemeyer et al., 2015), as illustrated by the significant transitions from requesting the second level of

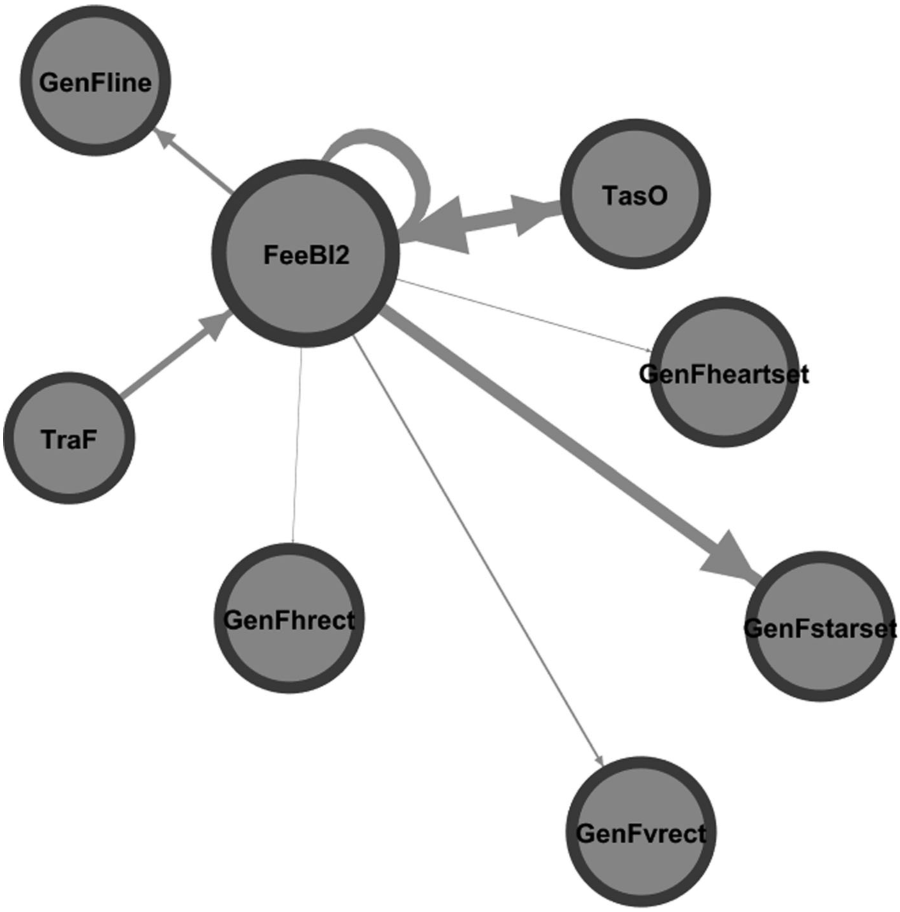

Figure 7. Significant transitions that relate to FeeBl2 activity.

feedback again and again, as well as by the fact that they switched frequently between opening the task and requesting the second-level feedback. These behaviour tendencies perhaps imply that students failed to obtain insights about the problem (Cox, 1996). Also, in a culture where the Confucian approach proposes that education is about cultivating oneself (Zhang, 2009), Chinese students are very keen to accomplish a task with as little help as possible. Furthermore, to be more effective in instruction, instructive feedback is used commonly in Chinese schools; Chinese students may not be very receptive to open and reflective feedback intended to encourage them to further explore to accomplish the task; instead they are more likely to be frustrated, confused, or bored if they are unable to complete the learning task after requesting feedback twice or more (Zhang, Gao, Mavrikis, Holmes, & Ma, 2019). With regard to this perspective, specific instructive feedback is more effective when students are confused (Grawemeyer et al., 2015).

This is further confirmed by the significant transitions identified in the third level of feedback. The findings showed that only after requesting the third-level feedback were students likely to use a different external representation to solve the task (ChaFS). Considering the effects of different levels of feedback, Grawemeyer et al. (2015) reported that instructive feedback appears to guide students to follow what needs to be done in the next step, whereas other types of feedback are less successful. Chinese students are more comfortable with instructive feedback, which stimulates them to build abstractions about the concept of fractions in their minds, under direct and explicit guidance.

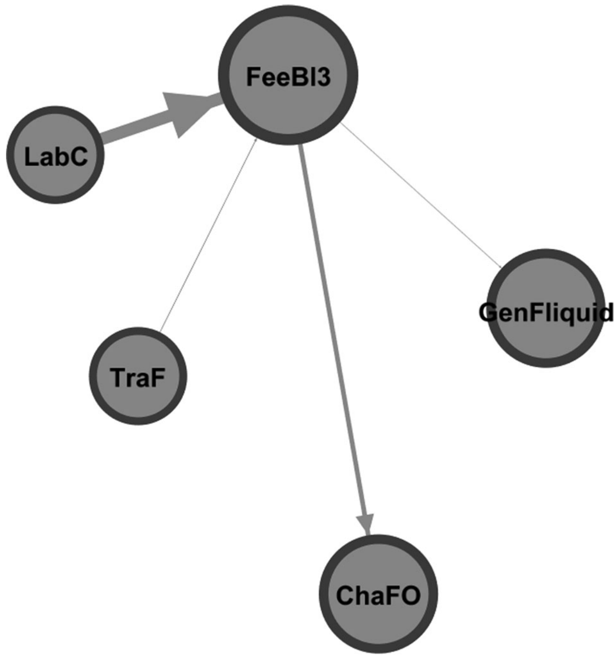

Figure 8. Significant transitions that relate to FeeBl3 activity.

Another important finding is that the students were more likely to generate new fractions or to open the task description again after receiving Socratic and guidance feedback. There were no significant transitions to other activities, such as changing the fraction values, using a different representation, computing fractions, or seeking scaffolding to help them solve the problem. Only after receiving the third-level feedback did the students change the fraction values with a different representation. The difficulties that students faced in utilising the first-level and second-level feedback are mainly attributed to the culture of viewing participation in learning tasks as being a competition to prove that one is intellectually elite; this deserves further investigation. It is undeniable that the eagerness to achieve good scores plays an important role in motivating students to undertake mathematics education seriously in China. However, simply taking the perspective of achieving a high score constrains the potential of feedback to use external representations to support exploratory learning. In addition, the findings also imply that serious consideration needs to be given to adapting prompts that have proved to be effective in Western countries to the local culture when introducing foreign-developed ITSs to China.

Conclusion

This study attempted to use pattern-finding analytical techniques (e.g. Amershi & Conati, 2009; Mavrikis, 2010) to improve our understanding of the impacts of feedback and choice of mathematical representations on learning fractions. Each of these important correlates of learning has been researched extensively (e.g. Ainsworth et al., 2002; Aleven et al., 2003; Porayska-Pomsta, Mavrikis, & Pain, 2008), but the outcomes remain conjectural. Learning analytics provide a unique perspective and can inform both future teaching practice in technology-mediated classrooms and system design. In this study, we analysed students' feedback interaction patterns and learning trajectories in choosing particular mathematical representations, and in working with these representations to perform a particular mathematical task. Given the limited scope of this research (the analysed data was generated by students each engaging with the Fractions Lab system for only approximately 45 min), we cannot generalise. Nevertheless, this work is novel in that it uses interaction patterns to gain insights into students' reasoning processes, and warrants further research. These patterns can then be used as visualisations or summarised insights for learning analytics tools for teachers (Mavrikis, Gutierrez-Santos, & Poulovassilis, 2016). Longitudinal studies using Fraction Labs with additional mathematics tasks and improved mechanisms of intelligent feedback, based on this first study, would also be welcome. Further investigation of how these interaction patterns might be applied to the Fractions Lab system in order to provide increasingly tailored support based on individual student needs and capabilities can further enhance students' technology-mediated learning experiences.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research was funded by the Data-informed Learning Design for Future Schools of Beijing Advanced Innovation Center for Future Education (AICFE), Beijing Normal University, China [grant number AICFE-IO-013]; 2019 Comprehensive Discipline Construction Fund of Faculty of Education, Beijing Normal University [grant number BNU-2019].

Notes on contributors

Dr Jingjing Zhang is Associate Professor, Big Data Centre for Technology-mediated Education, School of Educational Technology, Faculty of Education, Beijing Normal University, China. She is interested in how learning occurs between people engaging in communication that is situated in daily work environments online and offline.

Ming Gao is a doctoral student, Research Centre of Distance Education, School of Educational Technology, Faculty of Education, Beijing Normal University, China. His research interests include learning analytics and educational data mining.

Dr Wayne Holmes is a Lecturer (Assistant Professor), Learning Sciences and Innovation, The Open University, UK. He has been involved in education and education research for more than 25 years.He is leading several learning and innovation research projects.

Dr Manolis Mavrikis is Associate Professor, Learning Technologies at UCL Knowledge Lab. His research centres on designing evidence-based intelligent technologies that provide direct feedback to learners, and in employing learning analytics tounderstand learning.

Dr Ning Ma is Associate Professor, Advanced Innovation Center for Future Education, School of Educational Technology, Faculty of Education, Beijing Normal University, China. Her interests include effective ICT pedagogy, teacher professional development, one-to-one classroom culture construction, etc.

ORCID

Jingjing Zhang http://orcid.org/0000-0002-0584-534X
Ming Gao http://orcid.org/0000-0003-0129-703X
Wayne Holmes http://orcid.org/0000-0002-8352-1594
Manolis Mavrikis http://orcid.org/0000-0003-0575-0823

References

Ainsworth, S., Bibby, P., & Wood, D. (2002). Examining the effects of different multiple representational systems in learning primary mathematics. *The Journal of the Learning Sciences*, *11*(1), 25–61.

Aleven, V., Stahl, E., Schworm, S., Fischer, F., & Wallace, R. (2003). Help seeking and help design in interactive learning environments. *Review of Educational Research*, *73*(3), 277–320.

Alfieri, L., Brooks, P. J., Aldrich, N. J., & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology*, *103*(1), 1–18.

Amershi, S., & Conati, C. (2009). Combining unsupervised and supervised machine learning to build user models for exploratory learning environments. *Journal of Educational Data Mining*, *1*(1), 18–71.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China and the US. *Journal of Mathematics Teacher Education*, *7*(2), 145–172.
Azevedo, R. (2005). Computer environments as metacognitive tools for enhancing learning. *Educational Psychologist*, *40* (4), 193–197.
Bakeman, R., & Gottman, J. M. (1997). *Observing interaction: An introduction to sequential analysis*. Cambridge, MA: Cambridge University Press.
Baker, R. S., & Yacef, K. (2009). The state of educational data mining in 2009: A review and future visions. *JEDM Journal of Educational Data Mining*, *1*(1), 3–17.
Bienkowski, M., Feng, M., & Means, B. (2012). Enhancing teaching and learning through educational data mining and learning analytics: An issue brief. *US Department of Education, Office of Educational Technology*, *1*, 1–57.
Boylan, M., Wolstenholme, C., Maxwell, B., Jay, T., Stevens, A., & Demack, S. (2016). *Longitudinal evaluation of the mathematics teacher exchange, China-England: Interim research report*. Sheffield: Department for Education, Sheffield Hallam University.
Cai, J. (2000). Mathematical thinking involved in US and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking and Learning*, *2*(4), 309–340.
Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM*, *39*(5–6), 459–473.
Cai, J., & Silver, E. A. (1995). Solution processes and interpretations of solutions in solving a division-with-remainder story problem: Do Chinese and US students have similar difficulties? *Journal for Research in Mathematics Education*, *26*(5), 491–497.
Cai, J., & Wang, T. (2010). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education*, *13*(3), 265–287.
Carenini, G., Conati, C., Hoque, E., Steichen, B., Toker, D., & Enns, J. (2014). Highlighting interventions and user differences: Informing adaptive information visualization support. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems (CHI'14)* (pp. 1835–1844). New York: ACM.
Chang, K.-E., Chen, Y.-L., Lin, H.-Y., & Sung, Y.-T. (2008). Effects of learning support in simulation-based physics learning. *Computers & Education*, *51*(4), 1486–1498.
Chen, B., Resendes, M., Chai, C. S., & Hong, H. Y. (2017). Two tales of time: Uncovering the significance of sequential patterns among contribution types in knowledge-building discourse. *Interactive Learning Environments*, *25*(2), 162–175.
Cheng, Y. W., Wang, Y., Cheng, I. L., & Chen, N. S. (2019). An in-depth analysis of the interaction transitions in a collaborative augmented reality-based mathematic game. *Interactive Learning Environments*, 1–15.
Cox, R. (1996). *Analytical reasoning with multiple external representations*. Edinburgh: University of Edinburgh Press.
Cox, R. (1997). Representation interpretation versus representation construction: An ILE-based study using switchERII. In B. du Boulay & R. Mizoguchi (Eds.), *Artificial intelligence in education: Knowledge and media in learning systems (Proceedings of the 8th world conference of the artificial intelligence in education society)* (pp. 434–444). Amsterdam: IOS.
Davis, E. A. (2000). Scaffolding students' knowledge integration: Prompts for reflection in KIE. *International Journal of Science Education*, *22*(8), 819–837.
Davis, J. A. (1971). *Elementary survey analysis*. Englewood Cliffs, NJ: Prentice-Hall.
Dawson, S. (2010). 'Seeing' the learning community: An exploration of the development of a resource for monitoring online student networking. *British Journal of Educational Technology*, *41*(5), 736–752.
Ferguson, R. (2012). Learning analytics: Drivers, developments and challenges. *International Journal of Technology Enhanced Learning*, *4*(5/6), 304–317.
Grawemeyer, B., Mavrikis, M., Holmes, W., Gutierrez-Santos, S., Wiedmann, M., & Rummel, N. (2016, April). Affecting off-task behaviour: how affect-aware feedback can improve student learning. In *Proceedings of the sixth International Conference on Learning Analytics & Knowledge* (pp. 104–113). New York: ACM.
Grawemeyer, B., Mavrikis, M., Holmes, W., Gutiérrez-Santos, S., Wiedmann, M., & Rummel, N. (2017). Affective learning: Improving engagement and enhancing learning with affect-aware feedback. *User Modeling and User-Adapted Interaction*, *27*(1), 119–158.
Grawemeyer, B., Mavrikis, M., Holmes, W., Hansen, A., Loibl, K., & Gutiérrez-Santos, S. (2015). The impact of feedback on students' affective states. In *CEUR Workshop Proceedings*(Vol. 1432, pp. 4–13).
Gunter, P. L., Jack, S. L., Shores, R. E., Carrell, D. E., & Flowers, J. (1993). Lag sequential analysis as a tool for functional analysis of student disruptive behavior in classrooms. *Journal of Emotional and Behavioral Disorders*, *1*(3), 138–148.
Hansen, A., Mavrikis, M., & Geraniou, E. (2016). Supporting teachers' technological pedagogical content knowledge of fractions through co-designing a virtual manipulative. *Journal of Mathematics Teacher Education*, *19*(2–3), 205–226.
Holmes, W., Mavrikis, M., Hansen, A., & Grawemeyer, B. (2015). Purpose and level of feedback in an exploratory learning environment for fractions. In C. Conati, N. Heffernan, A. Mitrovic, & M. F. Verdejo (Eds.), *Artificial intelligence in education. Lecture notes in computer science* (pp. 620–623). Berlin: Springer.
Hou, H. T. (2012). Exploring the behavioral patterns of learners in an educational massively multiple online role-playing game (MMORPG). *Computers & Education*, *58*(4), 1225–1233.
Hou, H. T., Sung, Y. T., & Chang, K. E. (2009). Exploring the behavioral patterns of an online knowledge-sharing discussion activity among teachers with problem-solving strategy. *Teaching and Teacher Education*, *25*(1), 101–108.

Hwang, G. J., & Chen, C. H. (2017). Influences of an inquiry-based ubiquitous gaming design on students' learning achievements, motivation, behavioural patterns, and tendency towards critical thinking and problem solving. *British Journal of Educational Technology*, *48*(4), 950–971.

Jeong, A. C. (2003). The sequential analysis of group interaction and critical thinking in online. *The American Journal of Distance Education*, *17*(1), 25–43.

Johnson, A. M., Reisslein, J., & Reisslein, M. (2015). Transitional feedback schedules during computer-based problem-solving practice. *Computers & Education*, *81*, 270–280.

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, *38*(5), 1008–1022.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, *41*(2), 75–86.

Knight, S., Wise, A. F., & Chen, B. (2017). Time for change: Why learning analytics needs temporal analysis. *Journal of Learning Analytics*, *4*(3), 7–17.

Kong, S. C. (2008). The development of a cognitive tool for teaching and learning fractions in the mathematics classroom: A design-based study. *Computers & Education*, *51*(2), 886–899.

Lai, C. L., & Hwang, G. J. (2015). A spreadsheet-based visualized Mindtool for improving students' learning performance in identifying relationships between numerical variables. *Interactive Learning Environments*, *23*(2), 230–249.

Lan, Y. F., Tsai, P. W., Yang, S. H., & Hung, C. L. (2012). Comparing the social knowledge construction behavioral patterns of problem-based online asynchronous discussion in e/m-learning environments. *Computers & Education*, *59*(4), 1122–1135.

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, *11*(1), 65–100.

Li, N., Cao, Y. M., & Mok, I. A. C. (2016). A framework for teacher verbal feedback: Lessons from Chinese mathematics classrooms. *EURASIA: Journal of Mathematics, Science & Technology Education*, *12*(9), 2465–2480.

Lin, T. J., Duh, H. B. L., Li, N., Wang, H. Y., & Tsai, C. C. (2013). An investigation of learners' collaborative knowledge construction performances and behaviour patterns in an augmented reality simulation system. *Computers & Education*, *68*, 314–321.

Mavrikis, M. (2010). Modelling student interactions in intelligent learning environments: Constructing Bayesian networks from data. *International Journal on Artificial Intelligence Tools*, *19*(6), 733–753.

Mavrikis, M., Grawemeyer, B., Hansen, A., & Gutierrez-Santos, S. (2014). Exploring the potential of speech recognition to support problem solving and reflection. In *European Conference on Technology Enhanced Learning* (pp. 263–276). Berlin: Springer.

Mavrikis, M., Gutierrez-Santos, S., & Poulovassilis, A. (2016). Design and evaluation of teacher assistance tools for exploratory learning environments. In *Proceedings of the Sixth International Conference on Learning Analytics & Knowledge* (pp. 168–172). New York: ACM.

Mavrikis, M., Holmes, W., Zhang, J., & Ma, N. (2018). Fractions lab goes east: Learning and interaction with an exploratory learning environment in China. In Rosé C. Penstein et al. (Eds.), *Artificial intelligence in education* (pp. 209–214). AIED 2018. Lecture Notes in Computer Science (vol. 10948). Berlin: Springer.

McComas, J. J., Moore, T., Dahl, N., Hartman, E., Hoch, J., & Symons, F. (2009). Calculating contingencies in natural environments: Issues in the application of sequential analysis. *Journal of Applied Behaviour Analysis*, *42*(2), 413–423.

Mousavinasab, E., Zarifsanaiey, N., Niakan Kalhori, S. R., Rakhshan, M., Keikha, L., & Ghazi Saeedi, M. (2018). Intelligent tutoring systems: A systematic review of characteristics, applications, and evaluation methods. *Interactive Learning Environments*, 1–22.

Nazemi, K., Breyer, M., Stab, C., Burkhardt, D., & Fellner, D. W. (2010). Intelligent exploration system-an approach for user-centered exploratory learning. In *EDULEARN10 proceedings* (pp. 6476–6484). Valencia: IATED.

Porayska-Pomsta, K., Mavrikis, M., & Pain, H. (2008). Diagnosing and acting on student affect: The tutor's perspective. *User Modeling and User-Adapted Interaction*, *18*(1), 125–173.

Putnam, L. L. (1983). Small group work climates: A lag-sequential analysis of group interaction. *Small Group Behavior*, *14*(4), 465–494.

Pyatt, K., & Sims, R. (2012). Virtual and physical experimentation in inquiry-based science labs: Attitudes, performance and access. *Journal of Science Education and Technology*, *21*(1), 133–147.

Roll, I., Baker, R. S., Aleven, V., & Koedinger, K. R. (2014). On the benefits of seeking (and avoiding) help in online problem-solving environments. *Journal of the Learning Sciences*, *23*(4), 537–560.

Romero, C., Ventura, S., & García, E. (2008). Data mining in course management systems: Moodle case study and tutorial. *Computers & Education*, *51*(1), 368–384.

Rummel, N., Mavrikis, M., Wiedmann, M., Loibl, K., Mazziotti, C., Holmes, W., & Hansen, A. (2016). Combining exploratory learning with structured practice to foster conceptual and procedural fractions knowledge. In *Proceedings of the 12th International Conference of the Learning Sciences (ICLS 2016)* (pp. 58–65).

Sackett, G. P. (1978). *Observing behaviour: Theory and applications in mental retardation* (Vol. 1). Baltimore: University Park Press.

Scheiter, K., & Gerjets, P. (2007). Learner control in hypermedia environments. *Educational Psychology Review*, *19*(3), 285–307.

Siemens, G. (2013). Learning analytics: The emergence of a discipline. *American Behavioral Scientist*, *57*(10), 1380–1400.

Stenning, K. (2002). *Seeing reason: Image and language in learning to think*. New York: Oxford University Press.

Suthers, D. D. (2001). Towards a systematic study of representational guidance for collaborative learning discourse. *Journal of Universal Computer Science*, *7*(3), 254–277.

Wampold, B. E. (1992). The intensive examination of social interaction. In T. R. Kratochwill & J. R. Levin (Eds.), *Single-case research design and analysis: New directions for psychology and education* (pp. 93–131). Hillsdale, NJ: Erlbaum.

Wang, J., & Lin, E. (2005). Comparative studies on US and Chinese mathematics learning and the implications for standards-based mathematics teaching reform. *Educational Researcher*, *34*(5), 3–13.

Yang, X., Guo, X., & Yu, S. (2016). Student-generated content in college teaching: Content quality, behavioural pattern and learning performance. *Journal of Computer Assisted Learning*, *32*(1), 1–15.

Yang, X., Li, J., Guo, X., & Li, X. (2015). Group interactive network and behavioral patterns in online English-to-Chinese cooperative translation activity. *The Internet and Higher Education*, *25*, 28–36.

Yin, C. J., Uosaki, N., Chu, H. C., Hwang, G. J., Hwang, J. J., Hatono, I., & Tabata, Y. (2017, January). Learning behavioural pattern analysis based on students' logs in reading digital books. In *Proceedings of the 25th International Conference on Computers in Education* (pp. 549–557). Asia-Pacific Society for Computers in Education.

Zhang, J. (2002). Representations of health concepts: A cognitive perspective. *Journal of Biomedical Informatics*, *35*(1), 17–24.

Zhang, X. (2009). The current states of the mathematics curriculum reform in the Mainland China and some cultural analyzing. *Research in Mathematical Education*, *13*(2), 91–101.

Zhang, J., Gao, M., Mavrikis, M., Holmes, W., & Ma, N. (2019). *Use of feedback according to students' affective state during problem solving*. Companion Proceedings of the 9th International Conference on Learning Analytics & Knowledge (LAK19), Society for Learning Analytics Research (SoLAR).

Zhou, X., Li, M., Li, L., Zhang, Y., Cui, J., Liu, J., & Chen, C. (2018). The semantic system is involved in mathematical problem solving. *NeuroImage*, *166*, 360–370.